Lect 1 Maxwell's equations

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§: what we have known

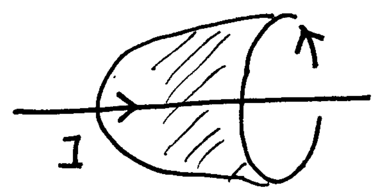
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\varepsilon_0} \quad \text{(Gauss's law)} \qquad \oint \vec{B}\cdot d\vec{a} = \ ?$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(Farady)} \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0 I + \ ?$$

Ampere's law

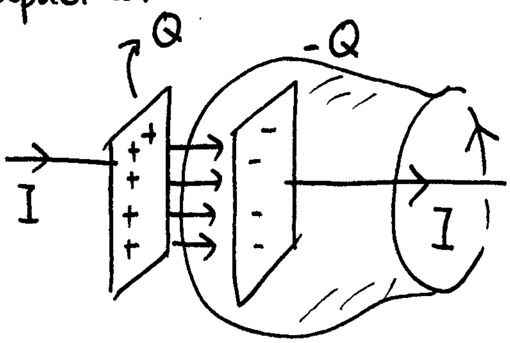
Farady's law: changing magnetic fields induce electric field.

Q: how about changing electric fields?



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ for steady current.

If we cut the ~~line~~ wire by a capacitor



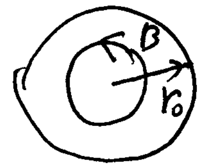
keep I steady, then $\frac{dQ}{dt} = I$. we expect that B field will be the same as before if the loop is far away from the capacitor.

∵ we can choose the surface to pass the inside of the capacitor,

There will be no current passing this surface any more ⇒

$$\oint \vec{B}\cdot d\vec{l} \neq 0 \ \cancel{\mu_0 I_{enclosed}}$$

(c) B lines form a circule due to ~~circle~~ circular sym



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi}{dt} = B\cdot 2\pi r$$

$$\mu_0\varepsilon_0 \frac{d}{dt}(\pi r^2 E) = B\cdot 2\pi r \Rightarrow B = \frac{\mu_0\varepsilon_0}{2} r \frac{dE}{dt} \quad \text{for } r < r_0$$

for $r > r_0$ $\quad B\cdot 2\pi r = \mu_0\varepsilon_0 \frac{d}{dt}(\pi r_0^2 E) \Rightarrow B = \frac{\mu_0\varepsilon_0 r_0^2}{2r}\frac{dE}{dt}$

B's maximal is at $r = r_0$ $\Rightarrow B_{max} = \frac{\mu_0\varepsilon_0 r_0}{2}\frac{dE}{dt}$

$= 1.2\times 10^{-4}\,T \sim 1.2$ Gauss.

Let's check for $r > r_0$ $\quad B = \frac{\mu_0\varepsilon_0}{2r} r_0^2 \frac{dE}{dt}$

$$\left\{ E = \frac{Q}{A\cdot\varepsilon_0} \Rightarrow \frac{dE}{dt} = \frac{dQ}{dt\,A\cdot\varepsilon_0} = \frac{I}{\pi r_0^2 \varepsilon_0} \right.$$

$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$

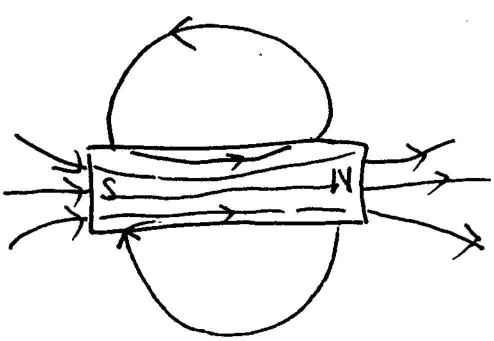
it looks as if it is produced by electric current.

§2. Gauss's law for magnetism.

$$\oint \vec{E}\cdot d\vec{A} = Q/\varepsilon_0 .$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

no magnetic charge



$$Q = CV = \left(\epsilon_0 \frac{A}{d}\right)(E \cdot d) = \epsilon_0 A E$$

$$\Rightarrow \frac{dQ}{dt} = I \Rightarrow \epsilon_0 A \frac{dE}{dt} = I$$

So we have $\oint \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 \epsilon_0 A \frac{dE}{dt} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

we need to combine these two contributions together

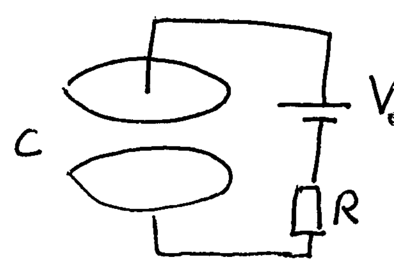
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \boxed{\epsilon_0 \frac{d\Phi_E}{dt}}$$

displacement current added by maxwell.

Ex

30 pF capacitor, circular plates of area $A = 100\ \text{cm}^2$

Charged by a 70 V battery through $2.0\,\Omega$ resistor.



Solution: RC circuit

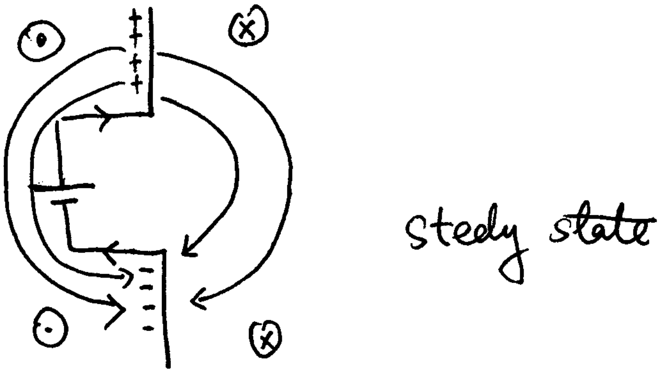
$$IR + \frac{Q}{C} = V_0 , \quad Q(t=0) = 0$$

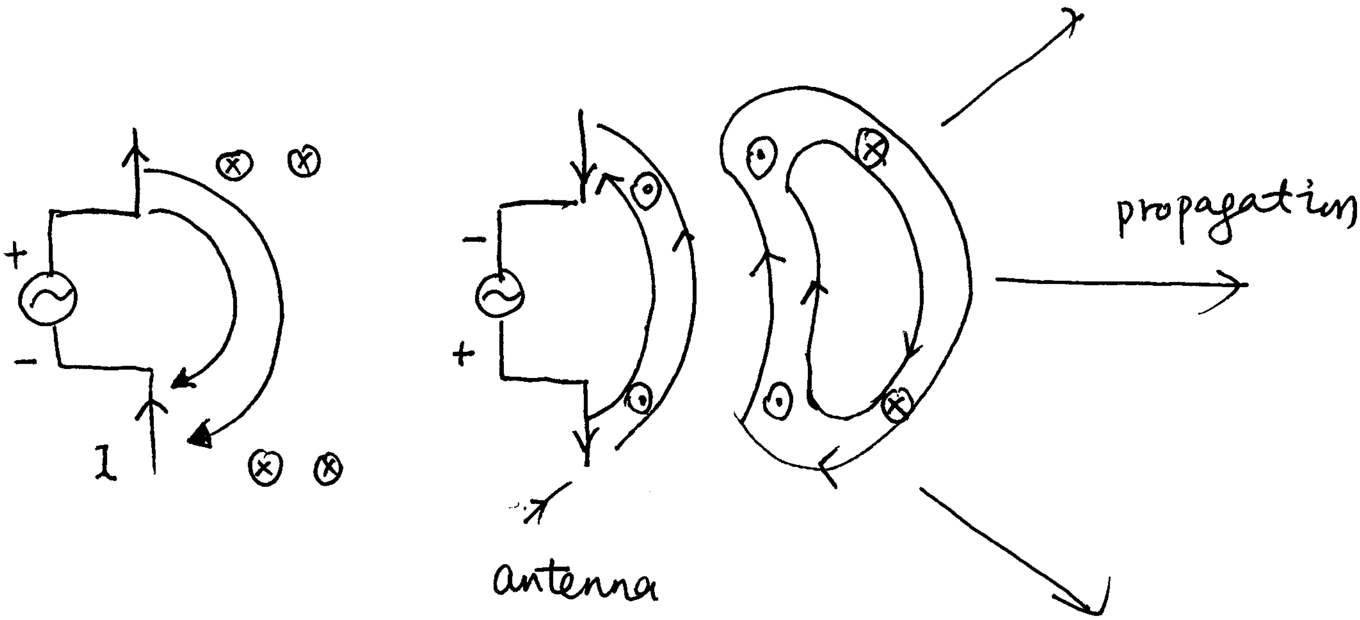
$$\frac{dQ}{dt} = -\frac{Q}{RC} + \frac{V_0}{R} \Rightarrow Q = CV_0\left[1 - e^{-t/RC}\right]$$

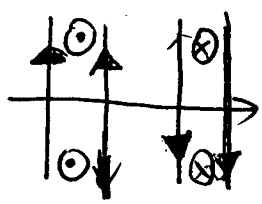
at $t = 0$. $\quad I = \frac{dQ}{dt} = \frac{V_0}{R} = \frac{70\text{V}}{2.0\,\Omega} = 35\text{A}$

the rate of change of E,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \Rightarrow \frac{dE}{dt} = \frac{dQ}{dt\, A\varepsilon_0} = 4.0 \times 10^{14}\ \text{V/m} \cdot \text{s}$$







wave front

E, B radiation field

$\propto \frac{1}{r}$ not $\frac{1}{r^2}$

The energy $\propto E^2 + B^2 \propto \frac{1}{r^2}$

far field

plane wave – transverse field

energy propagatin $\vec{S} = \vec{E} \times \vec{B}$

Lect 2 E-M waves, speed of light

differential form of maxwell equation

$$\nabla\cdot\vec{E} = \rho/\varepsilon_0 \qquad \nabla\cdot\vec{B}=0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}$$

in the free space where $\rho=0$, $\vec{j}=0$, we have $\begin{cases}\nabla\cdot\vec{E}=0\\ \nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}\end{cases}$

let us try plane wave solutions with $\vec{k}\,//\,\hat{x}$

$$\vec{E}(r,t) = \vec{E}_0\cos(kx-\omega t) \Rightarrow \vec{E}_0\cdot\hat{x}k=0 \Rightarrow \boxed{E_{0,x}=0} \quad \vec{E}\perp\vec{k}$$

$$\vec{B}(r,t) = \vec{B}_0\cos(kx-\omega t) \Rightarrow \vec{B}_0\cdot\hat{x}k=0 \Rightarrow \boxed{B_{0,x}=0} \quad \vec{B}\perp\vec{k}$$

E-M waves are transverse waves.

$$\nabla\times\vec{E} = (\partial_x E_y - \partial_y E_x)\hat{z} + (\partial_y E_z - \partial_z E_y)\hat{x} + (\partial_z E_x - \partial_x E_z)\hat{y}$$

$$= \partial_x E_y\,\hat{z} - \partial_x E_z\,\hat{y},$$

$$= (-k E_{0y}\hat{z} + k E_{0,z}\hat{y})\sin(kx-\omega t) = -k\,\hat{x}\times(E_{0,y}\hat{y} + E_{0,z}\hat{z})\ \sin(kx-\omega t)$$

$$= -\vec{k}\times\vec{E}_0\ \sin(kx-\omega t)$$

$$\frac{\partial\vec{B}}{\partial t} = \omega\vec{B}_0\ \sin(kx-\omega t) \Rightarrow \omega\vec{B}_0 = \vec{k}\times\vec{E}_0 \Rightarrow \boxed{\vec{B}_0 = \frac{1}{v}\hat{k}\times\vec{E}_0}$$

$$v = \frac{\omega}{k}$$

Similarly $\nabla \times \vec{B} = -\vec{k} \times \vec{B}_0 \sin(kx-\omega t)$

$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E} = \omega \mu_0 \varepsilon_0 \vec{E}_0 \sin(kx-\omega t)$$

$$\Rightarrow -\vec{k} \times \vec{B}_0 = \omega \mu \varepsilon_0 \vec{E}_0 \Rightarrow -\hat{k} \times \vec{B}_0 = v \mu \varepsilon_0 \vec{E}_0$$

$$\vec{B}_0 = \frac{1}{v} \hat{k} \times \vec{E}_0$$

$$\Rightarrow -\hat{k} \times (\hat{k} \times \vec{E}_0) = v^2 \mu \varepsilon_0 E_0 \Rightarrow E_0 = v^2 \mu \varepsilon_0 E_0 \Rightarrow v^2 = c^2 = \frac{1}{\sqrt{\mu \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

Example: $f = 60\text{ Hz}$, propagating along $\hat{z}$-direction

$\vec{E}$ along x-direction, $E_0 = 200\text{ V/m}$, Solve $\vec{E}(\vec{r}, t)$, $\vec{B}(\vec{r}, t)$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = 1.26 \times 10^{-6} \text{ m}^{-1}$$

$$\omega = 2\pi f = 377/s \qquad \Rightarrow \vec{E} = \vec{E}_0 \cos(kx - \omega t)$$

$$= 2\cos(1.26 \times 10^{-6} x - 377t)\, \hat{x} \text{ V/m}$$

$$\vec{B}_0 = \frac{1}{c} \hat{k} \times \vec{E}_0 = \frac{2\text{T}}{3 \times 10^8} \hat{z} \times \hat{x} = 6.67 \times 10^{-9} \hat{y}$$

$$\vec{B} = 6.67 \times 10^{-9} \text{T} \cdot \cos(1.26 \times 10^{-6} x - 377t)\, \hat{y}$$

light is an E-M wave

Radio wave: long, middle, short

micro-wave

infrared

visiable light < Red ⟷ purple: $4\times10^{14}\,Hz \sim 7.5\times10^{14}\,Hz$; $0.75\,\mu m$ ... $0.4\,\mu m$

ultra violet

x- rays

γ- rays

E-M wave can propagate in vaccum, It does need a media.

Example: $\lambda = \frac{c}{f}$: if $f = 60Hz \Rightarrow \lambda = 5\times10^{6}\,m = 5000\,km$

$93.3\,MHz \qquad \lambda = 3.22\,m$

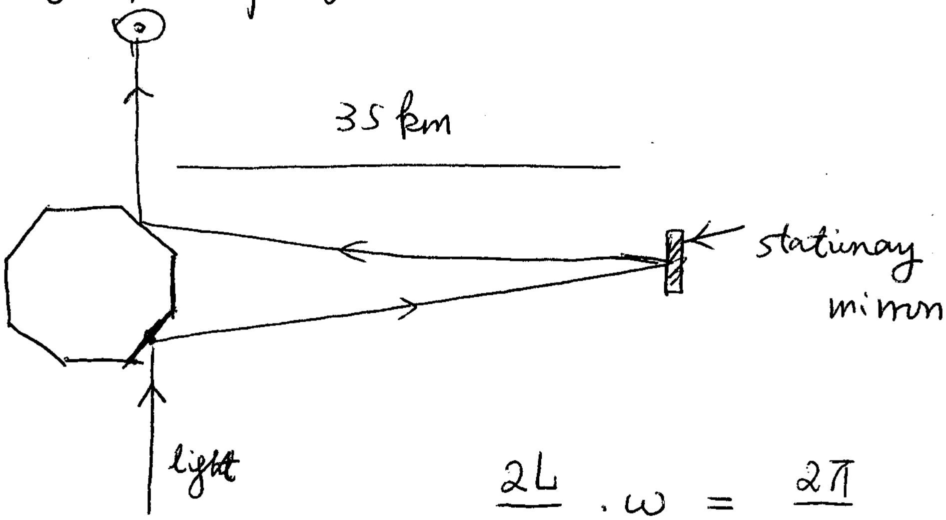
$4.74\times10^{14}\,Hz \qquad \lambda = 63\times10^{-7}\,m = 633\,nm.$

cellphone attenna $\quad \lambda = 4\cdot l \simeq 4\cdot 8.5\,cm \approx 0.34\,m$

$f = \frac{c}{\lambda} \approx 880\,MHz$

satelite phone delay $\quad t = \frac{2\times 36000\,km}{300000\,km/S} \approx \frac{7.2}{30}\,S = 0.24\,S$

Measuring speed of light



$$\frac{2L}{c}\cdot\omega_{min} = \frac{2\pi}{8}$$

$$\Rightarrow \omega_{min} = \frac{\pi}{8}\frac{c}{L} \qquad f = \frac{\omega}{2\pi} = \frac{c}{16L} = \frac{3\times10^{5}}{16\times35}\ /s$$

$$\approx 5\times10^{2}\ Hz$$

Lect 3. Energy in E-M waves, Poynting vector

energy density $\quad u_E = \frac{1}{2}\varepsilon_0 E^2$

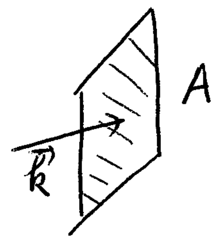
$$u_B = \frac{1}{2\mu_0} B^2$$

for E-M field $\quad u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 E^2 + \frac{(\mu_0\varepsilon_0)^2 B^2}{2\mu_0} = \varepsilon_0 E^2$

$$B = \mu_0\varepsilon_0 E$$

define Poynting vector — energy flow

$$S = \frac{dU}{A\,dt} = \frac{u\,dV}{A\cdot dt} = uc = \varepsilon_0 c E^2 = \frac{E\cdot B}{\mu_0}$$



direction $\vec{S} \parallel \vec{k}$

$$\Rightarrow \vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

$$\overline{E^2} = \frac{E_0^2}{2}$$

$$\Rightarrow \bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{E_0 B_0}{2\mu_0}$$

Example: E, B from the sun $\quad S = 1350\ W/m^2$

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 \Rightarrow E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = 1.01\times10^3\ V/m$$

$$B_0 = \frac{E_0}{c} = 3.37\times10^{-6}\ T$$

Radiation pressure

the relation between momentum and enegy of light

$$p = \frac{U}{c}$$

if the radiation is absorbed $\Rightarrow$ $\Delta p = \frac{\Delta U}{c}$

is bounced back $\Delta p = \frac{2\Delta U}{c}$

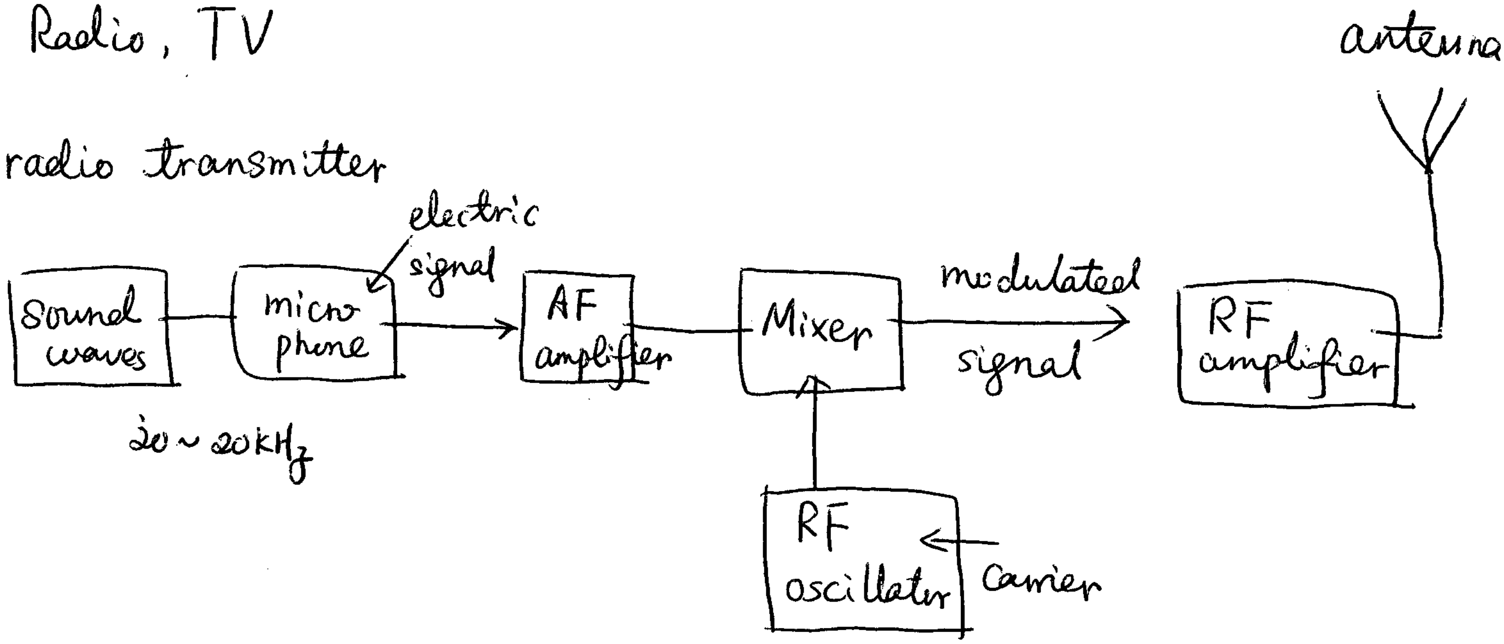
$$F = \frac{dp}{dt} \Rightarrow P = \frac{\Delta F}{\Delta A} = \frac{\Delta U}{c\,\Delta A\,\Delta t} = \begin{cases} \bar{S}/c & \text{absorbed} \\ 2\bar{S}/c & \text{bounced back} \end{cases}$$

sun light pressure : $\bar{S} = 1000\ W/m^2$

$$P = \bar{S}/c = 3\times10^{-6}\ N/m^2$$

§ Radio, TV

radio transmitter

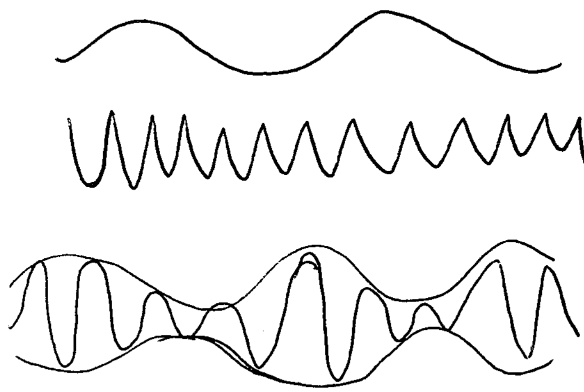


carrier frequency: AM $530\,kHz \sim 1700\,kHz$

FM $88\,MHz \sim 108\,MHz$

TV ~~100 MHz~~ a few hundred MHz

AM

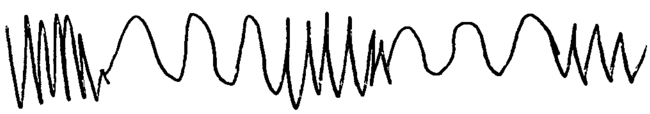


audio / $\cos(\omega_a t)$

carrier $A\cos(\omega_c t)$

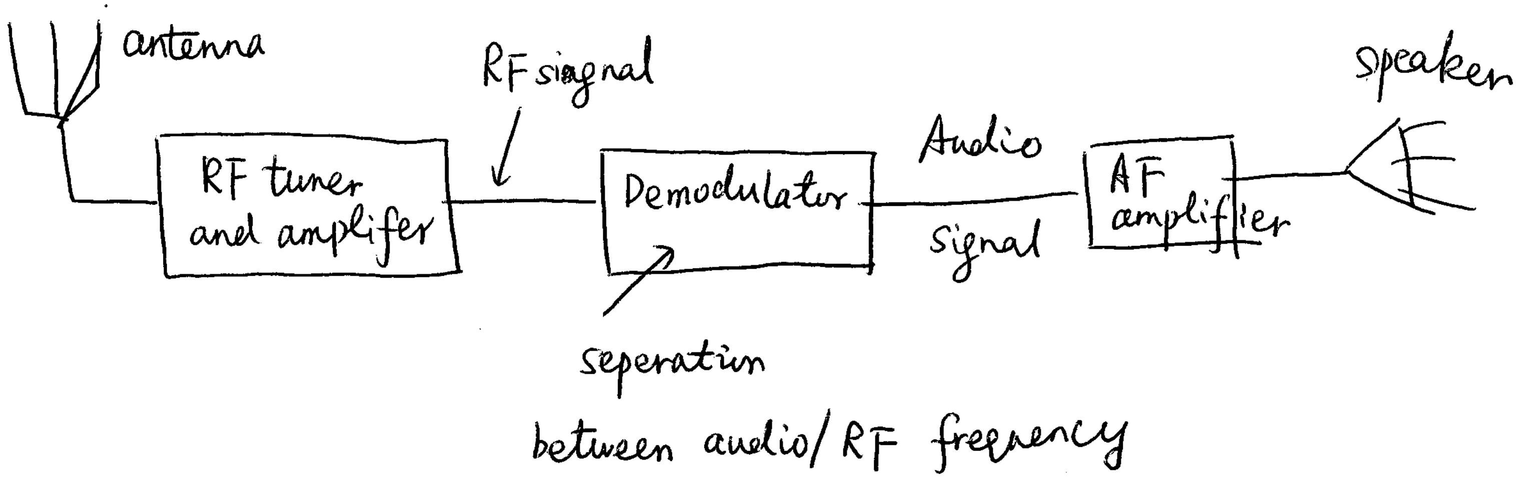
signal $\cos(\omega_a t)\cos(\omega_c t)$

FM

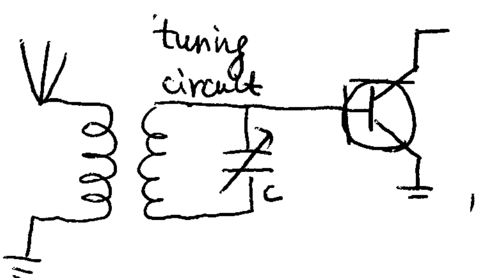


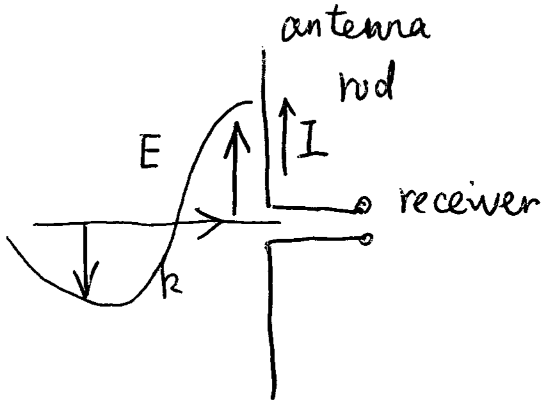
signal $\cos\{[\omega_c + a\cos(\omega_a t)]t\}$

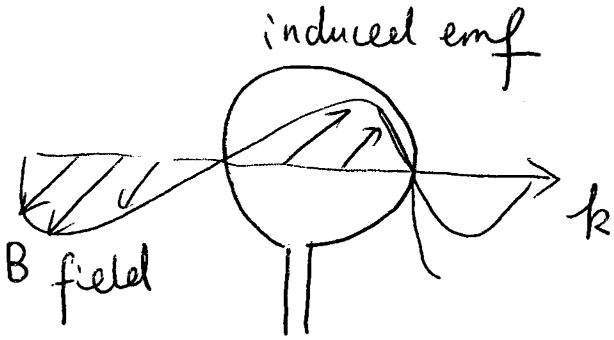
Receiver



Radio







100MHz FM , $\lambda = c/f \simeq 3\,m$

length of antenna $\frac{\lambda}{2}$, or $\frac{\lambda}{4}$