Phy 220 Group theory HW 2

Prob 1: Please prove a few statements below:

① The set of symmetry operations that commute with a Hamiltonian H form a group.

② If G is a set of elements forming a group, then AG is a rearrangement of this set if A is a group element, such that $\sum_{R\in G} f(R) = \sum_{R\in G} f(AR)$, where f is an arbitrary function.

③ If S is an element of group G, and Y is any object whose multiplication with group elements R is defined then,

$$A = \sum_{R\in G} R Y R^{-1}$$, commutes with any element $\in G$,

i.e $[S, A] = 0$.

④ The Dirac character Ω_c of a class is defined as the sum of of the elements in that class,

$$\Omega_c \equiv \sum_{T\in C} T.$$ Prove that $$\Omega_c = \frac{n_c}{|G|} \sum_{R\in G} R\, u\, R^{-1}$$

where u is an element in the class, $|G|$ is the order of the group.

Prob 2:

Prove two lemmas of Schur which are given in the lecture notes.

Prob 3:

Prove any linear representation of a finite group is equivalent to an unitary representation.

Prob 4: Prove that the characters of an irredicible representation

$$\frac{1}{|G|}\sum_{g\in G}\chi(g^2) = \begin{cases} 1 & \text{real representation} \\ -1 & \text{pseudo real Rep} \\ 0 & \text{complex representations.} \end{cases}$$

Prob 5: (a) For 3D rotation group SO(3), figure out its class structure.

(b) Consider the double group of SO(3), i.e. SU(2), figure out its class structure.

(c) Figure out a convenient rule for constructing the class structure of a point group, (a discrete subgroup of SO(3)).

(d) Can you generalize it to the double point group?

Hint: Consider rotations at the same rotation angles but at different directions.