Phy-220 Group theory final

Problem 1: Cubic anisotropy

Symmetry analysis can help to build up field theory for the system you want to study. Now let me consider to write down a free energy for a magnetic system, which is a functional of magnetic denisity $\vec{m}(\vec{r})$ denoted as $F[\vec{m}(\vec{r})]$. Let us

($\vec{m}(\vec{r})$: distribution)

expand $F[\vec{m}(\vec{r})]$ in terms of polynomials, and only consider the homogenous distibution, i.e. $\vec{m}(\vec{r}) = \vec{m}$ which is a const over space. Then

$$F(m) = a_i m_i + a_{ij} m_i m_j + a_{ijk} m_i m_j m_k + a_{ijkl} m_i m_j m_l m_k + \cdots, \quad (i,j,k,l = x, y, z)$$

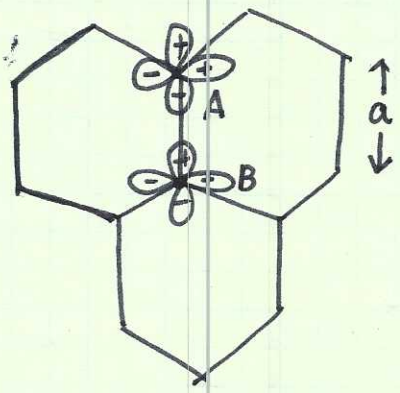
and we truncate at the quartic level.

① For an isotropic space, figure out the structures of the coeffients a_i, a_{ij}, a_{ijk}, a_{ijkl}.

Hint, $F(m)$ needs to be invariant under all the symmetries of an isotropic space, and also possibly other symmetries due to physical requirment such as time-reversal symmetry.

(2) If the system does not have all the symmetric of a free space. but has the cubic symmetry O. Try to do ~~problem~~ part (1) again.

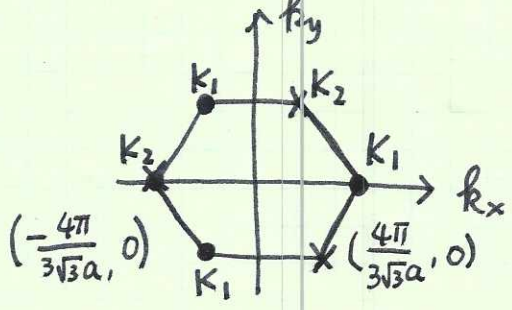
Problem 2: p-orbital band ~~structure~~ in the honeycomb lattice.



(2) One each site, there are a pair of degenerate P_x and P_y-orbitals. Each unit cell has a pair of sites A and B.

The Brouillone zone is plotted below.

~~The honeycomb lattice has the~~ ~~tion~~



(1) What's the group of wavevector K_1? You only need to figure out the ~~rotation~~ part of point group.

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(2) There are four bases for the group of wavevector K_1.

$$\psi_{A\,P_{x,y}}(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{r}_{A^0}} e^{i\vec{K}_1\cdot\vec{r}}\, \phi_{P_{x,y}}(\vec{r} - \vec{r}_{A^0})$$

$$\psi_{B,\,P_{x,y}}(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{r}_{B^0}} e^{-i\vec{K}\cdot\vec{r}}\, \phi_{P_{x,y}}(\vec{r} - \vec{r}_{B^0})$$

Basing on this set of basis, decompose them into irreducible representations of the group of wavevector $G_{\vec{K}_1}$. Can you understand them in terms of angular momentum numbers? Can you compare with the case of graphene?

③ open part: for extra credit. You may further consider to add spin-orbit coupling, and try to figure out more physics.

Problem 3: Cnside a 3D dimond lattice. Let us use the convention that forom each A site there are four bonds towarding it's four B-neighbours along (111), $(\bar{1}\bar{1}1)$, $(\bar{1}1\bar{1})$ and $(1\bar{1}\bar{1})$. Figure out the elements of its space group.

Is it symmorphic or non-symmorphic?