HW 2

9.22: Set S_0 the inertial frame, the Eq of motion reads

$$m\left(\frac{d^2r}{dt^2}\right)_{S_0} = -\frac{qQ}{r^2}\hat{r} + q\left(\frac{d\vec{r}}{dt}\right)_{S_0} \times \vec{B}$$

now let us change to a frame S rotating with angular velocity $\vec{\Omega}$ relative to S_0. Then LHS

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{S_0} = m\left(\frac{d^2\vec{r}}{dt}\right)_S - 2m\left(\frac{d\vec{r}}{dt}\right)_S \times \vec{\Omega} + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$$

$$\text{RHS} \Rightarrow -\frac{qQ}{r^2}\hat{r} + q\left[\left(\frac{d\vec{r}}{dt}\right)_S + \vec{\Omega}\times\vec{r}\right]\times\vec{B}$$

Let us choose $\vec{\Omega} = -\dfrac{q\vec{B}}{2m}$, then $\left(\frac{d\vec{r}}{dt}\right)_S$ in the ~~the~~ Crolices force (LHS) and Lorentz force (RHS) cancels

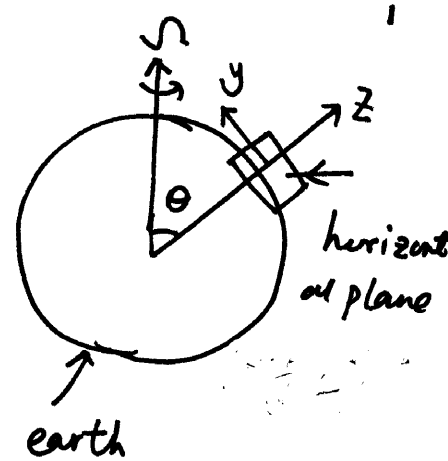
$$\Rightarrow m\left(\frac{d^2\vec{r}}{dt}\right)_S - \frac{q^2}{4m}(\vec{B}\times\vec{r})\times\vec{B} = -\frac{qQ}{r^2}\hat{r} - \frac{q^2}{2m}(\vec{B}\times\vec{r})\times\vec{B}$$

$$\Rightarrow m\left(\frac{d^2\vec{r}}{dt}\right)_S = -\frac{qQ}{r^2}\hat{r} - \frac{q^2}{4m}(\vec{B}\times\vec{r})\times\vec{B}$$

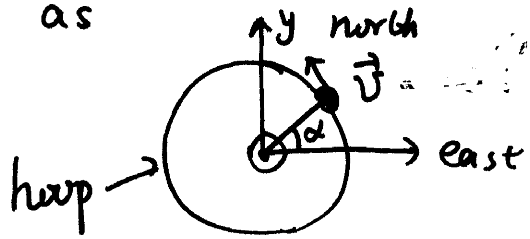
if B is weak, we can drop the second term, then the motion is in an inverse square force. Thus in "S", the motion is ellipse /hypenbola and in "S_0", the ellipse precesses.

9.30

The ~~?~~ configuration of the hoop is plotted



as



The hoop is put in the xy (east-north) horizontal plane. It is spining around the z-axis. Pick a small segment from $\alpha \to \alpha$ to

$$\Rightarrow \quad dm = \frac{m}{2\pi} d\alpha$$

$$d\vec{F}_{cor} = 2dm \; \vec{v} \times \vec{\Omega}, \qquad \vec{v} = \omega r(-\sin\alpha, \cos\alpha, 0)$$

$$\vec{\Omega} = \Omega(0, \sin\theta, \cos\theta)$$

$$\Rightarrow \cancel{d\vec{F}_{cor} =}$$

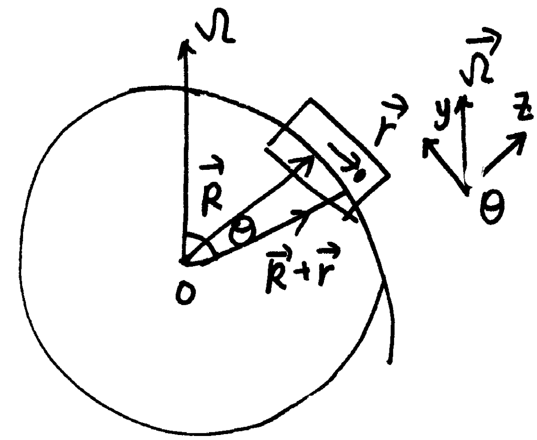
$$d\vec{P} = \vec{r} \times d\vec{F}_{cor} = 2dm \; \vec{r} \times (\vec{v} \times \vec{\Omega})$$

$$= 2dm \left[\vec{v}(\vec{r}\cdot\vec{\Omega}) - \vec{\Omega}(\vec{r}\cdot\vec{v})\right] = 2\,dm\,\sin\theta\,\omega r^2 \Omega(-\sin^2\alpha, \sin\alpha\cos\alpha, 0)$$

$$\vec{P} = \int d\vec{P}$$

$$\Rightarrow \quad \vec{P} = -2\sin\theta\,\omega r^2\Omega\, m \int_0^{2\pi} \frac{\sin^2\alpha \, d\alpha}{2\pi}\,\hat{x} = -m\omega r^2 \Omega \sin\theta\,\hat{x}$$

$\vec{P}$ along $\hat{y}, \hat{z}$ are zero.



9.34: The puck's position at $\vec{R}+\vec{r}$ respect to the center of the earch. $\vec{R}$ is the center of the platform, and $\vec{r}$ is respect to the center of the platform. $\vec{R} \perp \vec{r}$.

$$m\ddot{\vec{r}} = m\vec{g}_0(r) + 2m\dot{\vec{r}}\times\vec{\Omega} + m[\vec{\Omega}\times(\vec{r}+\vec{R})]\times\vec{\Omega} + \vec{F}_{other}$$

$$\vec{g}_0(r) = -GM\frac{\vec{R}+\vec{r}}{|\vec{R}+\vec{r}|^3} = -GM\frac{\vec{R}+\vec{r}}{(R^2+r^2)^{3/2}} \approx -GM\frac{\vec{R}+\vec{r}}{R^3}\left(1-\frac{3r^2}{2R^2}\right)$$

$$\approx -GM\frac{\vec{R}}{R^3} - \frac{GM\vec{r}}{R^3} = \vec{g}_0(0) + g_0(0)\frac{\vec{r}}{R}$$

The centrifugal force $= m(\vec{\Omega}\times\vec{R})\times\vec{\Omega} + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$

The first term combines with $\vec{g}_0(0)$ as the actural free-fall acceleration $\vec{g}(0)$. The second term $\underset{\vec{F}'}{\underset{\parallel}{m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}}}$ can be dropped which will be justified later.

$\Rightarrow$ The equation of motion

$$\ddot{\vec{r}} = \vec{g}(0) - g\,\vec{r}/R + 2\dot{\vec{r}}\times\vec{\Omega} + \frac{\vec{F}_{other}}{m}$$

Considering the in-plane motion, and remember

$\vec{r} = (x, y, 0)$. $\vec{\Omega} = \Omega[0, \sin\theta, \cos\theta]$

$$\Rightarrow \begin{cases} \ddot{x} = -\dfrac{gx}{R} + 2\dot{y}\Omega\cos\theta \\ \ddot{y} = -\dfrac{gy}{R} - 2\dot{x}\Omega\cos\theta \end{cases}$$

define $\omega_0 = \sqrt{g/R} = 1.24 \times 10^{-3}\,s^{-1}$

where $R = 6400\,km$

This is the same equation of Foucault Penduleum.

$$\Omega_0 = \frac{2\pi}{24\text{ hours}} \approx 7 \times 10^{-5}\,s^{-1}, \qquad \Rightarrow \omega_0 \gg \Omega$$

$\Rightarrow$ the puck oscillates with ω_0, and precesses with $\Omega_z = \Omega\cos\theta$

The ~~centr~~ second term of the centrifugal force we droped

$F' = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$

$\sim m\Omega^2 A$ where A is the amplitude

$$F_{cr} \sim m\Omega v \sim m\Omega\omega_0 A$$

$$\Rightarrow \frac{F'}{F_{cr}} \sim \frac{\Omega}{\omega_0} \ll 1$$

and the gravity restoring force $\dfrac{mgr}{R} \sim mA\omega_0^2 \gg F_{cr}, F'$.

so $\vec{F}'$ can indeed be dropped.