Prob 1)

① $$\vec{F} = e\vec{E} - \frac{m\vec{v}}{\tau} = m\dot{\vec{v}}$$

$$\Rightarrow \quad \dot{v} = \frac{eE}{m} - \frac{v}{\tau} \qquad \Rightarrow \frac{v_\infty}{\tau} = \frac{eE}{m} \quad \text{with } v_\infty = \frac{eE\tau}{m}$$

$$\Rightarrow \quad v(t) = A e^{-t/\tau} + v_\infty$$

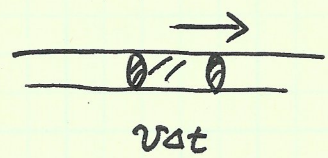
$$\Rightarrow \quad v(t) = (v_0 - v_\infty) e^{-t/\tau} + v_\infty ,$$

v_∞ does not depend on v_0

③ Consider the long time limit $t \to \infty$, $v(t) \longrightarrow v_\infty$.

then $$j = ne v_\infty = \frac{e^2 n E \tau}{m}$$

$$\Rightarrow \boxed{\sigma = \frac{ne^2\tau}{m}}$$

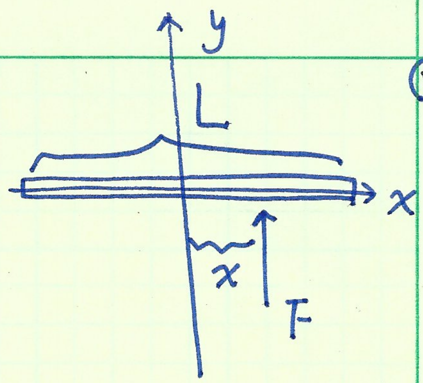


② $$I = \frac{\Delta Q}{\Delta t} = \frac{en v \Delta t A}{\Delta t} = en v A$$

$$j = I/A = en v$$

at $t \gg \tau$, j becomes steady, hence $v = v_\infty$

$$j = en v_\infty = \frac{e^2 n E \tau}{m}$$



Prob 3)

① $\vec{P} = F\Delta t\, \hat{y}$

$\Rightarrow M\vec{v}_{cm} = \vec{P} \Rightarrow \vec{v}_{cm} = \frac{F\Delta t}{M}\hat{y}$

② in the center of mass frame, the Rod beames to rotate around the z-axis.

$$\frac{d\vec{L}_{cm}}{dt} = \vec{\Gamma}_{cm}, \qquad \vec{\Gamma}_{cm} = \vec{r}\times\vec{F} = xF\,\hat{z}$$

The moment of inertial in the center of mass frame

$$I = \int_{-L/2}^{L/2} \rho x^2\, dx = \frac{1}{3}\rho x^3 \Big|_{-L/2}^{L/2} = \frac{2}{3}\rho\left(\frac{L}{2}\right)^3 = \frac{1}{12}\rho L^3$$

$$M = \int \rho\, dx = \rho L$$

$$\Rightarrow I = \frac{1}{12}ML^2$$

$$\Rightarrow I\omega = \Gamma_{cm}\Delta t \Rightarrow \omega = \frac{12xF\Delta t}{ML^2}\hat{z}$$

③ ~~Left end~~ $\cancel{\vec{v}_L = \vec{v}_{cm} - \omega\frac{L}{2} + \vec{\omega}\times\vec{r}}$

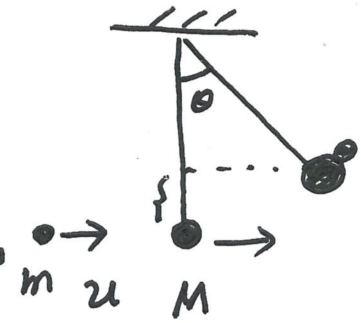
$\Rightarrow \vec{v}_L$ and $\vec{v}_R$ are along $\hat{y}$-direction in the Lab frame.

in the CM frame $\quad \vec{v}_{L,cm} = -\hat{y}\,\omega\frac{L}{2} = -\hat{y}\,\frac{6xF\Delta t}{ML}$

$$\vec{v}_{R,cm} = \hat{y}\,\omega\frac{L}{2} = \hat{y}\,\frac{6xF\Delta t}{ML}$$

$$\vec{v}_L = \vec{v}_{cm} + \vec{v}_{L,cm} = \frac{F\Delta t}{M}\left(1 - \frac{6x}{L}\right)\hat{y}$$

$$\vec{v}_R = \vec{v}_{cm} + \vec{v}_{R,cm} = \frac{F\Delta t}{M}\left(1 + \frac{6x}{L}\right)\hat{y}$$



Prob 3): during the collision, the pendulum does not have time to move, There's no horizontal external force. Hence the momentum along the horizontal direction is conserved.

$$mu = (m+M)v_0 \Rightarrow v_0 = \frac{m}{m+M}u$$

· After the collision, the mechanical energy is conserved.

$$E_K = \frac{1}{2}(m+M)v_0^2, \quad E_G = (M+m)gl(1-\cos\theta)$$

$\Rightarrow$ At the $\theta = \theta_m$, $E_K = 0$

$$(M+m)gl(1-\cos\theta_m) = \frac{1}{2}(m+M)v_0^2$$

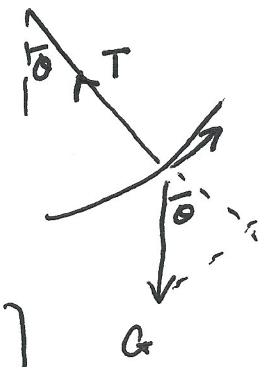
$$1-\cos\theta_m = \frac{v_0^2}{2gl} \Rightarrow \cos\theta_m = 1 - \frac{u^2}{2gl}\left(\frac{m}{m+M}\right)^2$$

$$\theta_m = \cos^{-1}\left[1 - \frac{u^2}{2gl}\left(\frac{m}{m+M}\right)^2\right]$$

b) at a general angle θ

$$(m+M)gl(1-\cos\theta) + \frac{1}{2}(m+M)v^2 = \frac{1}{2}(m+M)v_0^2$$

$$v^2 = v_0^2 - 2gl(1-\cos\theta) = \frac{m^2u^2}{(m+M)^2} - 2gl(1-\cos\theta)$$



$$T - G\cos\theta = (M+m)v^2/l$$

$$T = (M+m)g\cos\theta + \frac{(M+m)}{l}\left[v_0^2 - 2gl(1-\cos\theta)\right]$$

$$= (M+m)g\left[-2+3\cos\theta\right] + \frac{m}{M+m}\frac{mu^2}{l}$$