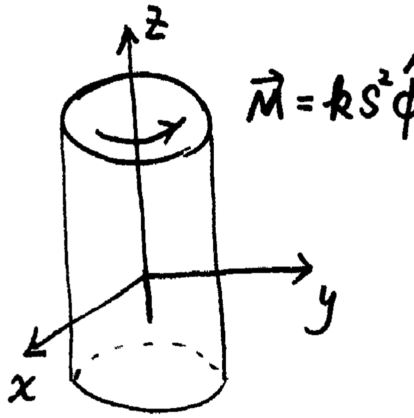
Solution HW 4 Phy 100B



6.8: a) using $\nabla \times \vec{H} = \frac{4\pi}{c} \vec{j}_{free} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$

$$= 0.$$

$$\vec{H} = \vec{B} - 4\pi \vec{M}.$$

there's no free current in this system $\Rightarrow \vec{H} = 0$ everywell. due to cylindrical symmetry, $\vec{H}$ has to be along circumferetial direction.

outside $\vec{H} = \vec{B} = 0$. Inside $\vec{H} = \vec{B} - 4\pi \vec{M} = 0 \Rightarrow \vec{B} = 4\pi k s^2 \hat{\phi}$.

or use $\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \nabla \times \vec{M}$

$$j_b = c \nabla \times \vec{M} = \frac{c}{s}\frac{\partial}{\partial s}(s k s^2)\hat{z} = c 3 k s \hat{z}$$

$$K_b = cM \times \hat{n} = -c k R^2 \hat{z}.$$

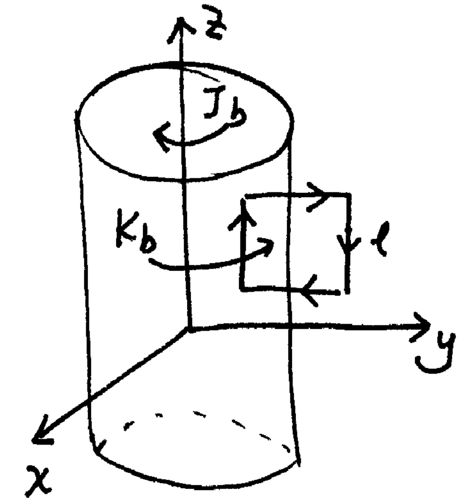
using Ampere's law for inside $B_\phi \cdot 2\pi \cdot s = \frac{4\pi}{c} c \int_0^s j_b \, s \, ds$

$$= 4\pi \, 3k \, \frac{s^3}{3} \cdot 2\pi$$

$$\Rightarrow \vec{B} = 4\pi k s^2 \hat{\phi} = 4\pi \vec{M} \quad (\text{for } r < R)$$

outside $\Rightarrow K_b \cdot 2\pi R + \int_0^R j_b \, s \, ds$

$= -ck \, 2\pi R^3 \hat{z} + c \cdot 3k \cdot 2\pi \frac{R^3}{3} \hat{z} = 0 \Rightarrow$ the current penetrating the loop is zero $\Rightarrow \vec{B} = 0$ for $r > R$.



6.12

a) $M = ks\hat{z}$. $\vec{J}_b = c\,\nabla\times\vec{M} = -ck\,\hat{\phi}$

$\vec{K}_b = c\vec{M}\times\hat{n} = ckR\,\hat{\phi}$

By the symmetry structure, we explained before $\vec{B}$ should be along the $\hat{z}$-direction.

$B_z = 0$ outside.

For B inside. $\Rightarrow B_z\cdot\ell = \frac{4\pi\ell}{c}[\,ckR\cdot \; - c\int_s^R k \;\; ds\,]$

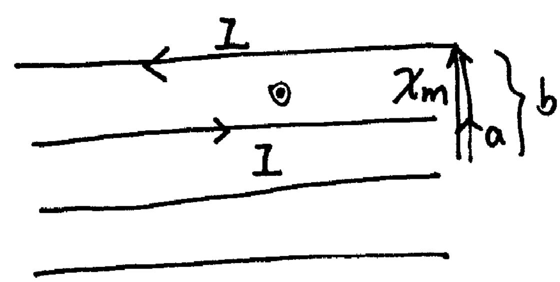
$= \quad 4\pi\ell k\,[R-(R-s)] = 4\pi\ell k s$

$\Rightarrow \vec{B}(s) = 4\pi k s\,\hat{z}$ inside

b) $\vec{H}$ should be along the $\hat{z}$-direction. By symmetry $\oint\vec{H}\cdot d\vec{\ell} = 0$

$\Rightarrow H = 0$. outside $\Rightarrow \vec{B} = \vec{H} = 0$

inside $\Rightarrow \vec{B} = \vec{H} + 4\pi\vec{M} = 4\pi\vec{M} = 4\pi ks\,\hat{z}$



6.16 a) between tubes, $\vec{H}$ is along $\hat{e}_\phi$

$\oint H\cdot d\ell = \frac{4\pi}{c}I$

$\Rightarrow H 2\pi s = \frac{4\pi}{c}I \Rightarrow \vec{H} = \frac{2}{sc}I\,\hat{e}_\phi \Rightarrow \vec{B} = \vec{H} + 4\pi\vec{M} = (1+4\pi\chi_m)\vec{H}$

$\Rightarrow \vec{B} = (1+4\pi\chi_m)\frac{2I}{sc}\,\hat{e}_\phi$

$$\vec{M} = \chi_m \vec{H} = \frac{2\chi_m I}{sc}\,\hat{e}_\phi \Rightarrow \vec{J}_b = c\,\nabla\times\vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(s\,\frac{2\chi_m I}{s}\right)\hat{z} = 0$$

$$K_b = c\vec{M}\times\hat{n} = \begin{cases} \frac{2\chi_m I}{a}\,\hat{z} & \text{at inner surface} \\ -\frac{2\chi_m I}{b}\,\hat{z} & \text{at outer surface} \end{cases}$$

or using $\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}[I_f + I_b]$

$$\Rightarrow B\cdot 2\pi s = \frac{4\pi}{c}[I + 2\pi a K_b] = 4\pi[I + 4\pi\chi_m I]$$

$$\Rightarrow B = \frac{2I}{sc}[1 + 4\pi\chi_m]\,\hat{e}_\phi$$