Lect 7 Electric field

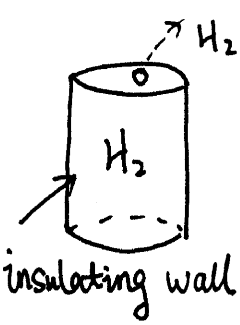
§1. Charge :

① conservation of charges : the total charge of an isolated system does not change. Charge cannot be created, and cannot be destroyed!

② invariance of charges : the above law is valid for any frame. In other words, the total charge of an isolated system is a relativistic invariant.

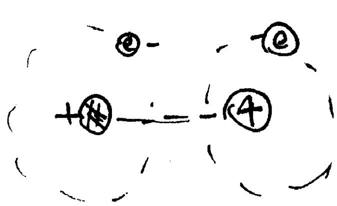
③ charge quantization : $-e$ — charge of an electron
$+e$ — charge of a proton

if charge isn't quantized, then it's difficult to explain why H-atom is neutral up to very high precision. Experiments show that Cs atom the total charge $< 10^{-16} e$. There are also experiment of H_2 molecule.



H_2 leaks out of the hole. If proton and electron charges are not exactly cancelled, the leaking of H_2 atom will change the total charge and its electric potential. $\Rightarrow < 10^{-20} e$

Inside H_2 atom, the motion of protons and electrons are so different

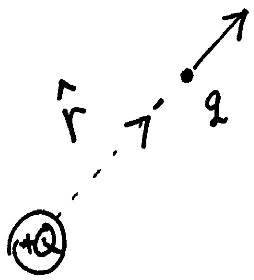


electrons move much faster that protons $\frac{v_e}{v_p} \simeq \frac{m_e}{m_p} \sim \frac{1}{2000}$.

so ~~charge~~ the value of is independant of mortion.

§2 Coulomb's law

Can you explain why the force is along the radial direction?



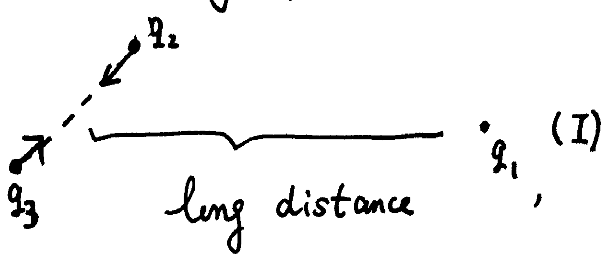
$$F = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}$$ — SI unit

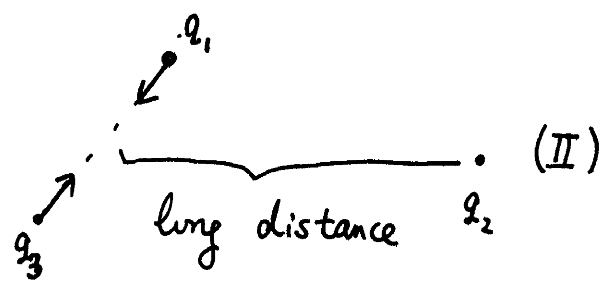
$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m}$

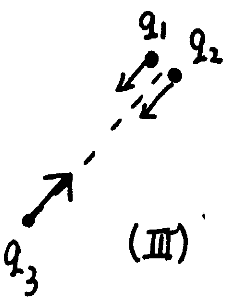
$$F = \frac{qQ}{r^2}$$ — Gauss unit

Physicist's preference.

The addibility of charge







the force in III on q_3 = the sum of those in I and II

The force between two charges is independent of the existance of the third charge.

liner superposition principle

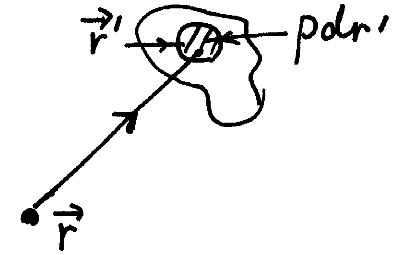
If we have a set of charges $q_1, \cdots q_n$ and a ~~test~~ charge Q, the force on Q

$$\vec{F} = Q\left[\frac{q_1(\vec{r}_Q - \vec{r}_1)}{|\vec{r}_1 - \vec{r}_Q|^3} + \frac{q_2(\vec{r}_Q - \vec{r}_2)}{|\vec{r}_2 - \vec{r}_Q|^3} + \cdots + \frac{q_n(\vec{r}_Q - \vec{r}_n)}{|\vec{r}_Q - \vec{r}_n|^3}\right]$$

define electric field

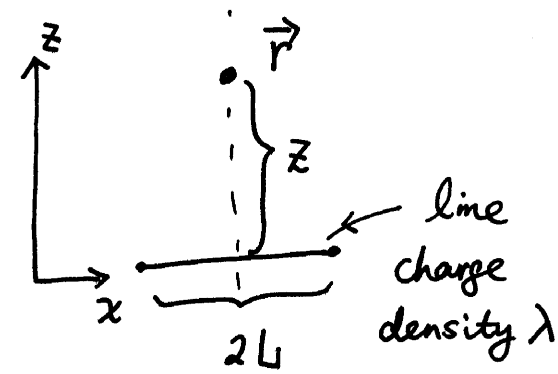
$$\boxed{\vec{E} = \vec{F}/Q = \sum_i \frac{q_i(\vec{r}_Q - \vec{r}_i)}{|\vec{r}_Q - \vec{r}_i|^3}}$$

§ Continuous charge distributions

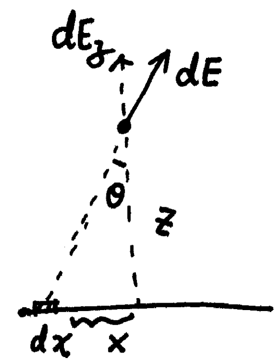


$$\vec{E}(\vec{r}) = \int \frac{(\vec{r}-\vec{r}')\rho\, d\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

Ex: ① symmetry analysis: Electric field at $\vec{r}$ (above the middle) can only along the z-axis. — can you explain?



② $$dE_z = \frac{\lambda dx}{(z^2+x^2)} \cdot \frac{z}{\sqrt{z^2+x^2}}$$



$$\Rightarrow E_z = \int_{-L}^{L} dx \frac{\lambda z}{(z^2+x^2)^{3/2}} = 2\int_0^L dx \frac{\lambda z^{-2}}{((\frac{x}{z})^2+1)^{3/2}}$$

$$\int \frac{dx}{(\sqrt{x^2+1})^3} = \frac{x}{\sqrt{x^2+1}}$$

$$\Rightarrow E_z = \frac{2\lambda}{z}\int_0^L d\left(\frac{x}{z}\right)\frac{1}{\left(\sqrt{(\frac{x}{z})^2+1}\right)^3} = \frac{2\lambda}{z}\left(\frac{x/z}{[(\frac{x}{z})^2+1]^{1/2}}\right)\Bigg|_0^L = \frac{2\lambda}{z}\frac{L/z}{((\frac{L}{z})^2+1)^{1/2}}$$

$$= \frac{2\lambda}{z}\frac{L}{\sqrt{L^2+z^2}}$$

if $z \gg L \Rightarrow E_z = \frac{2\lambda L}{z^2}$

if $z \ll L \Rightarrow E_z = \frac{2\lambda}{z}$, which is independent of L.