# Novel Sp(2N) and SU(2N) quantum magnetism and Mott physics – large spins are different

Congjun Wu

Department of Physics, University of California, San Diego

Current work:

1. Z. C. Zhou, Z. Cai, C. Wu, Y. Wang, Phys. Rev. B 90, 235139 (2014) .
2. D. Wang, Y. Li, Z. Cai, Z. Zhou, Y. Wang, C. Wu, Phys. Rev. Lett. 112, 156403 (2014).
3. Z. Cai, H. Hung, L. Wang, D. Zheng, C. Wu, Phys. Rev. Lett. 110, 220401 (2013) .
4. C. Wu, Nature Physics 8, 784 (2012) (News and Views).

Earlier work:

1. C. Wu, J. P. Hu, and S. C. Zhang, Phys. Rev. Lett. 91, 186402 (2003).
2. C. Wu, Phys. Rev. Lett. 95, 266404 (2005),
3. C. Wu, Mod. Phys. Lett. B 20, 1707 (2006) (brief review).

Nov 16, 2015, UBC

# Current collaborators

| | |
|---|---|
| Shenglong Xu | (UCSD) |
| Da Wang | (UCSD→ Nanjing Univ.) |
| Yi Li | (UCSD→ Princeton) |
| Zi Cai | (UCSD→ Innsbruck) |
| Hsiang-hsuan Hung | (UCSD→UIUC→ UT Austin) |
| Yu Wang, Zhi-Chao Zhou | (Wuhan Univ.) |

Collaborators on earlier works: S. C. Zhang (Stanford), J. P. Hu (Purdue), S. Chen and Y. P. Wang (IOP, CAS).

Acknowledgments: A. L. Fetter, E. Fradkin , T. L. Ho, J. Hirsch, D. Arovas, Y. Takahashi, F. Zhou.

Supported by NSF, AFOSR.

# Outline

- **Introduction: what is large?**

  Large symmetry (large N) rather than large spin magnitude (large S).
  Quantum spin fluctuations are enhanced rather than suppressed.

- Generic Sp(4) symmetry (spin-$\frac{3}{2}$).

  Unification of antiferromagnetism, superconductivity, and charge-density-wave.

  http://online.kitp.ucsb.edu/online/coldatoms07/wu2/

- Slater v.s. Mott: quantum phase transition at SU(6) -- QMC

  Interplay between charge and spin degrees of freedom

- Pomeranchuk effect (thermodynamics) -- QMC.

# The simplest interacting model of lattice fermions

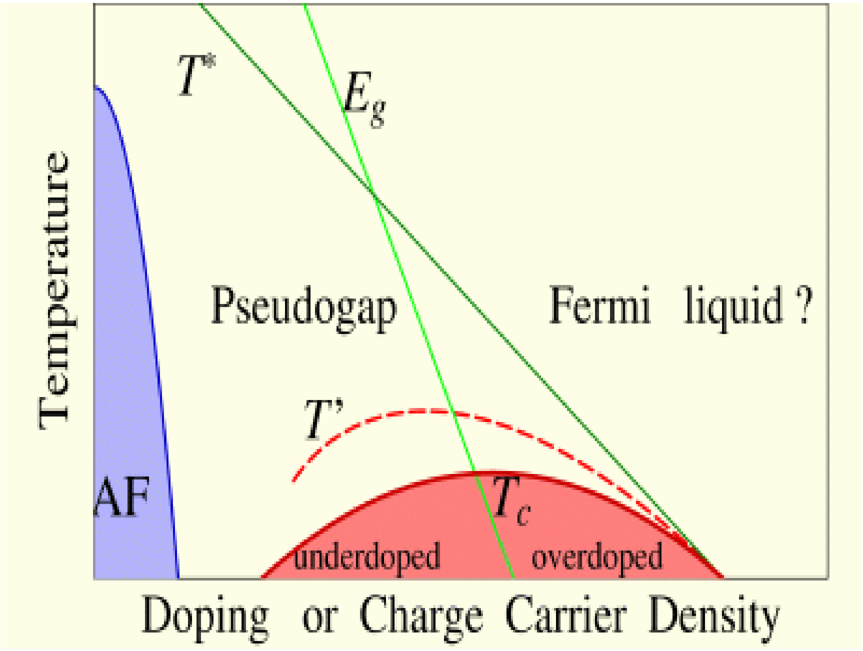
$$H = -\sum_{\langle ij\rangle,\sigma} t\{c^{+}_{i,\sigma}c_{j,\sigma} + h.c.\} - \mu\sum_{i,\sigma} c^{+}_{i,\sigma}c_{i,\sigma} + U\sum_{i} n_{i,\uparrow}n_{i,\downarrow}$$

- Hubbard 1963: itinerant ferromagnetsim (FM), not successful.

- But successful for metal-Mott insulator transitions.

- Can the single band Hubbard describe high $T_c$ cuprates?

--- Still in debates.



# Some rigorous results

- **1D Mott physics**: half-filled (U>0).

1. Charge gap opens at infinitesimal U (relevance of Umklapp term)
2. Spin channel remains critical – no symmetry breaking

C. N. Yang, PRL 19, 1312 (1967); Lieb and F. Y. Wu, PRL 20, 1445, (1968).

Field theoretical methods, DMRG simulations

- **2D AFM long-range-order:** the square lattice (half-filled).

Determinant quantum Monte-Carlo (DQMC): Sign-problem free at half filling -- non-perturbative method, asymptotically exact

Blackenbecler, Scalapinio, Sugar, PRD (1981); J. Hirsch, PRB 31, 4403 (1985).

# Hidden symmetry (pseudo-spin)

- Yang and Zhang's $\eta$-pairing → generators of SU(2) in the charge channel.

$$\eta^- = \sum_i (-)^i c_{i\downarrow} c_{i\uparrow}, \quad \eta^+ = \sum_i (-)^i c^+_{i\uparrow} c^+_{\downarrow}, \quad [\eta^-, \eta^+] = 2N$$

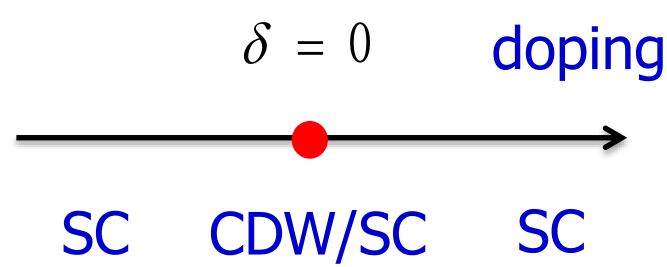
- Degeneracy between charge-density-wave (CDW) and superconductivity (SC) at half-filling (U<0)

$$O_{cdw} = \sum_i (-)^i n_i, \; \Delta = \sum_i c_{i\uparrow} c_{i\downarrow}, \; \Delta^+ = \sum_i c^+_{i\downarrow} c^+_{i\uparrow} \qquad [\eta^+, \Delta] = O_{CDW}$$

- Pseudo-Goldstone: $\eta$-mode

$$H(\eta^+ | G_{SC}\rangle) = (\mu - \mu_0)(\eta^+ | G_{SC}\rangle),$$

$$(\mu \geq \mu_0)$$



# Exotic spin states in the Mott-insulating phase

- Bosonic large-*N* -- Neel, dimer ordering.

  Arovas, Auerbach PRB1988, Sachdev and Read, Nucl. Phy. B 1989.

- Fermionic large-*N* -- spinon Fermi surface, Dirac point, etc.

  Affleck and Maston PRB1988, Hermele et al PRB2004, Lee, Nagaosa, Wen, RMP2006

- RVB, quantum dimer model, etc.

  Anderson 1973; Rokhsar, Kivelson PRL1988; Fradkin, Kivelson Mod. Phys. Lett 1990; Moessner and Sondhi PRL 2001.

- Frustration -- ring exchange, $J_1$-$J_2$ square lattice, Kagome, etc.

  Jiang, Fisher, Sheng, Motrunich et al 2008-2012; Jiang, Yao, Balents PRB 2012; Yan, Huse, White Science 2011.

# Theory progress with large-spin fermions

- Novel physics inaccessible in usual solid state systems.

- Early work by Ho and Yip (PRA and PRL 1999).

Richer Fermi liquid properties and Cooper pairing structures than those in spin-1/2 electron systems.

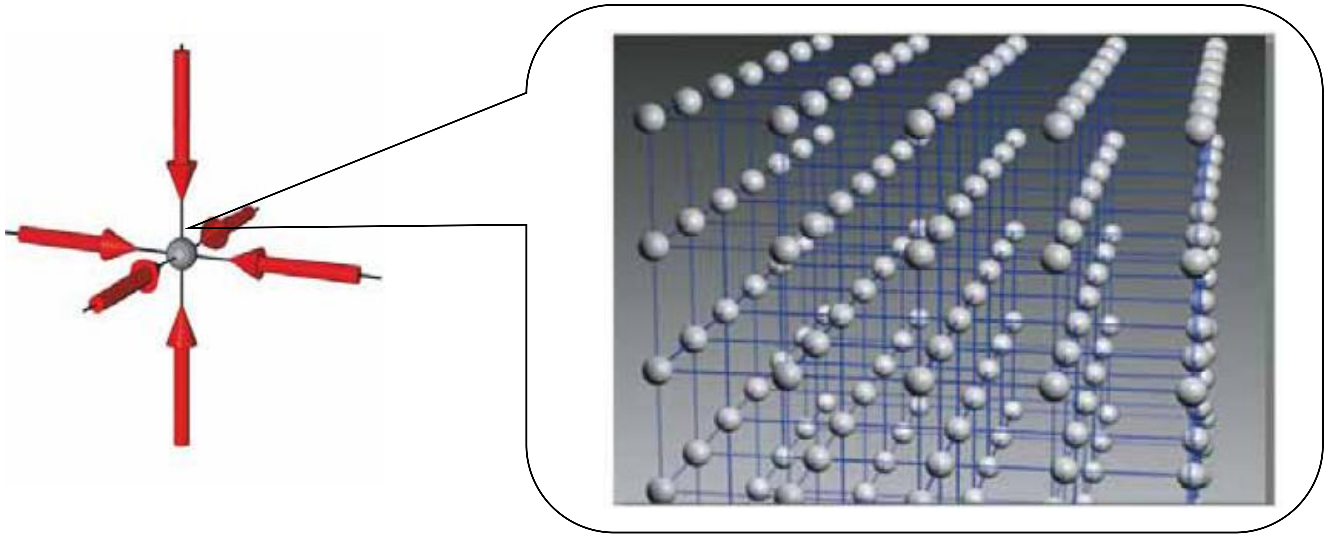
- **A new view point: high symmetries, Sp(2N), SU(2N).**

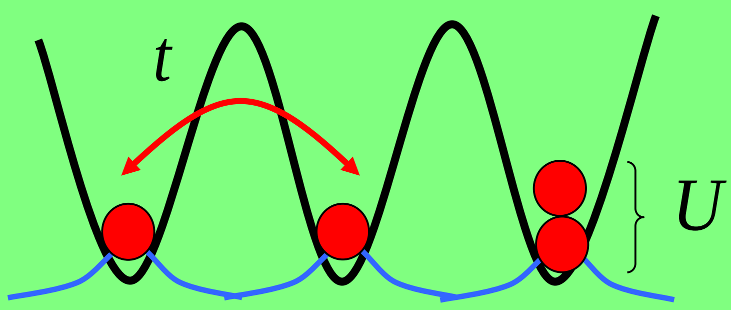
Sp(4), SO(5) ,SU(4) : (**spin -$\frac{3}{2}$**) **$^{132}$Cs, $^{9}$Be, $^{135}$Ba, $^{137}$Ba, $^{201}$Hg**
C. Wu, S. C. Zhang, S. Chen, Y. P. Wang, A. Tsvelik, G. M. Zhang, Lu Yu, X. W. Guan, Azaria, Lecheminant, et al. (2003 ---).

SU(2N): V. Guriare, M. Hermele, A. Rey, E. Demler, M. Lukin, P. Zoller, et al. (2010 ---).

# A new strongly correlated system: optical lattices

- Interaction effects tunable by varying laser intensity.






**t : inter-site tunneling**
**U: on-site interaction**

# Experiment breakthrough of large-spin fermions

Selected for a Viewpoint in *Physics*

90401 (2010) PHYSICAL REVIEW LETTERS wee 5 NOVE

PRL 105, 190401 (2010)

**Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas**

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

)2 (2010) PHYSICAL REVIEW LETTERS

PRL 105, 030402 (2010)

**Degenerate Fermi Gas of $^{87}$Sr**

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

## Viewpoint

Physics 3, 92(2010)

**Exotic many-body physics with large-spin Fermi gases**

**Congjun Wu**
*Department of Physics, University of California, San Diego, CA 92093, USA*

Published November 1, 2010

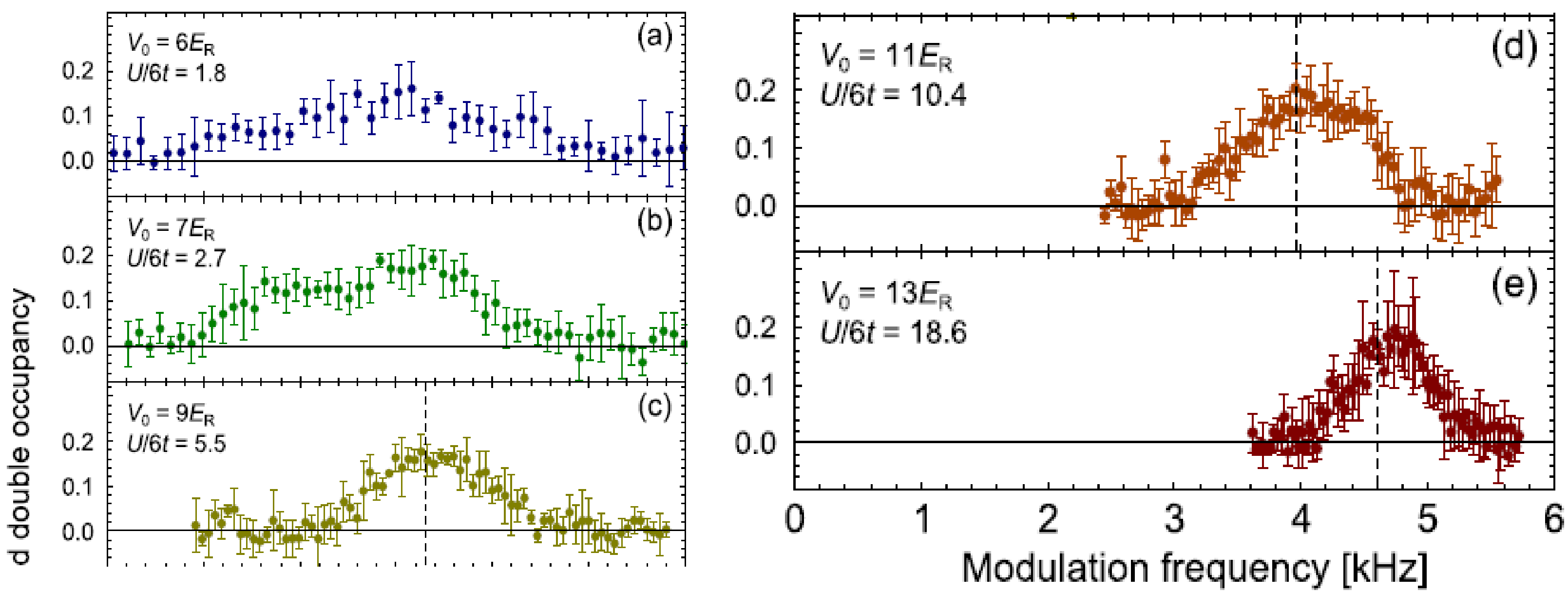
*The experimental realization of quantum degenerate cold Fermi gases with large hyperfine spins opens up a new opportunity for exotic many-body physics.*

nature physics
ARTICLES
PUBLISHED ONLINE: 23 SEPTEMBER 2012 | DOI: 10.1038/NPHYS2430

# An SU(6) Mott insulator of an atomic Fermi gas realized by large-spin Pomeranchuk cooling

**S. Taie,et.al, Nature phys. 8, 825(2012).**

Shintaro Taie[1*], Rekishu Yamazaki[1,2], Seiji Sugawa[1] and Yoshiro Takahashi[1,2]




- Many recent progresses: Fallani et al; Jun Ye et al; K. Sengstock et al; Foelling/Bloch et al, …….

# What is large?

- High symmetry (large N, SU(2N), Sp(2N)) rather than large spin magnitude (large S).

• High symmetries do not occur frequently in nature, since every new symmetry brings with itself a possibility of new physics, they deserve attention.

--- comment from D. Controzzi and A. M. Tsvelik, cond-mat/0510505

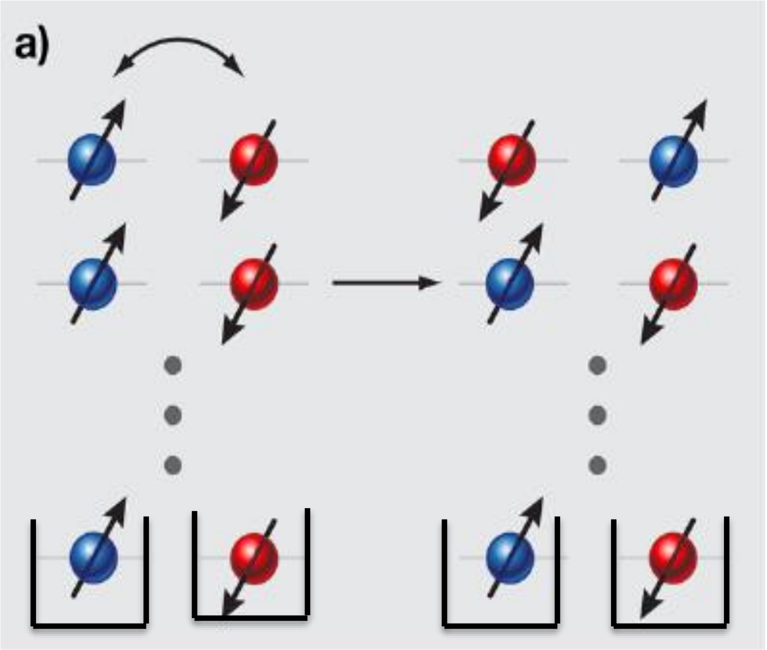
- Quantum spin fluctuations are enhanced NOT suppressed.

• SU(2N) and Sp(2N) were introduced to condensed matter physics as a formal tool, say, 1/N-expansion.

# Transition metal oxides **(large S → classical)**

- **Large spin magnitude** from Hund's coupling.
- Inter-site coupling: exchange **a single pair** of electrons.
- **1/S-fluctuations**: $\Delta S_z = \pm 1$
- Bilinear exchange dominates

$$\frac{t^2}{U}\vec{S}_i\cdot\vec{S}_j + \frac{t^4}{U^3}(\vec{S}_i\cdot\vec{S}_j)^2 + \ldots$$

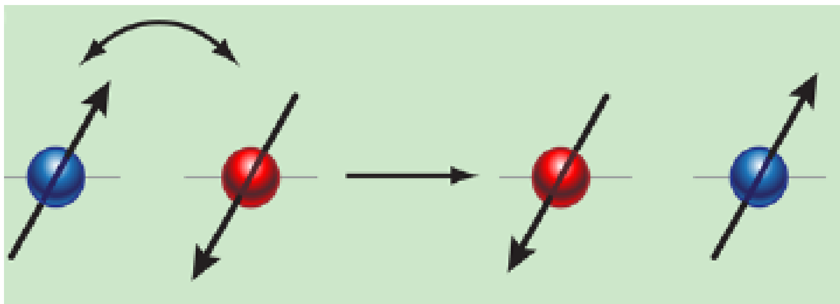



C. Wu, Physics 3, 92 (2010).

C. Wu, Nature Physics 8, 784 (2012) (News and Views).

# Cold fermions: **large N→ enhanced fluctuations!**

- Large-hyperfine-spin as a whole object (no ionization).



$$\Delta S_z = \pm 1, \pm 2, \ldots \pm S$$

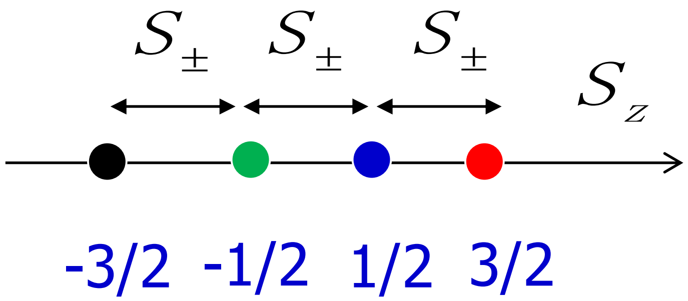
- One step of super-exchange can completely overturn spin config.

- Bilinear, bi-qudratic, bi-cubic terms, etc., are all at equal importance.

$$\vec{S}_i \cdot \vec{S}_j, (\vec{S}_i \cdot \vec{S}_j)^2, (\vec{S}_i \cdot \vec{S}_j)^3$$

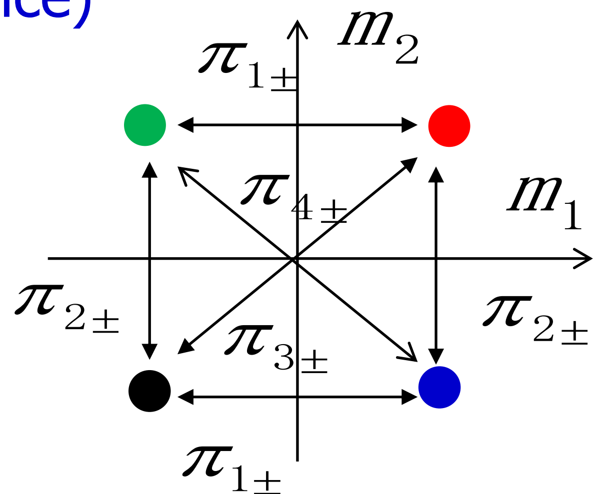
# Two views of spin quartet (weight diagrams of Lie algebra)

Solid: SU(2) (1D lattice)




- A high rank spinor Rep. of a small group.
- Off-diagonal operator: (fluctuation) $S_\pm$

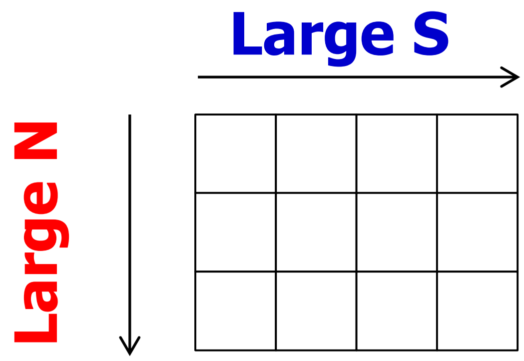
Cold fermions Sp(4) or SO(5) (2D lattice)




- The fundamental spinor Rep of a large group.
- Much more off-diagonal operators.

$$\pi_{1\pm},\ \pi_{2\pm},\ \pi_{3\pm},\ \pi_{4\pm}$$

# SU(2N), Sp(2N) (2N=2S+1)




- Alkaline-earth fermions: SU(2N), equivalent 2N components.

  fully filled electronics shells → spin-independent interaction

- Alkali fermions: broken SU(2N), spin-dependent interaction.

- Symplectic symmetry:

  SU(2N) → Sp(2N)

  Good properties under time-reversal transformation.

# Outline

- Introduction: what is large?    large N v.s. large S

- **Generic Sp(4) symmetry (spin-3/2).**

  Unification of antiferromagnetism, superconductivity, and charge-density-wave.

  http://online.kitp.ucsb.edu/online/coldatoms07/wu2/

# The simplest case spin-3/2: **Hidden symmetry**!

- Spin 3/2 atoms: $^{132}$Cs, $^{9}$Be, $^{135}$Ba, $^{137}$Ba, $^{201}$Hg.

- **Sp(4) (SO(5))** symmetry without fine tuning regardless of dimensionality, particle density, and lattice geometry!

Sp(4) in spin 3/2 systems ←→ SU(2) in spin ½ systems

- SU(4) symmetry is realized iff the interaction is spin-independent.

- Importance of high symmetries: unification of competing orders, description of strong spin fluctuations, etc.



C. Wu et al. Phys. Rev. Lett. 91, 186402 (2003).

# Spin-3/2 Hubbard model in optical lattices

$$H = \sum_{\langle ij\rangle,\alpha} -t\{c^{+}_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_{i} c^{+}_{i,\alpha} c_{i,\alpha}$$
$$+ U_0 \sum_{i} \eta^{+}(i)\eta(i) + U_2 \sum_{a=1\sim 5} \chi^{+}_{a}(i)\chi_{a}(i)$$

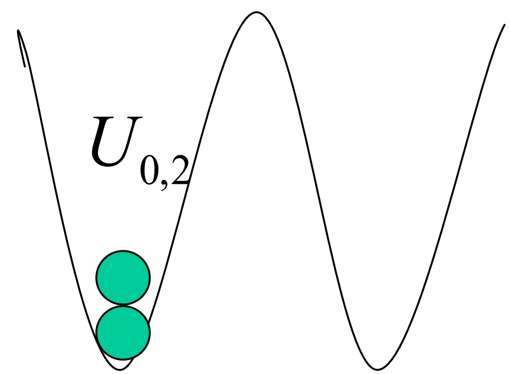
$\Uparrow \left|\frac{3}{2}\right\rangle \quad \uparrow \left|\frac{1}{2}\right\rangle$

$\downarrow \left|-\frac{1}{2}\right\rangle \quad \Downarrow \left|-\frac{3}{2}\right\rangle$

- Fermi statistics: only $F_{tot}$=0, 2 are allowed; $F_{tot}$=1, 3 are forbidden.

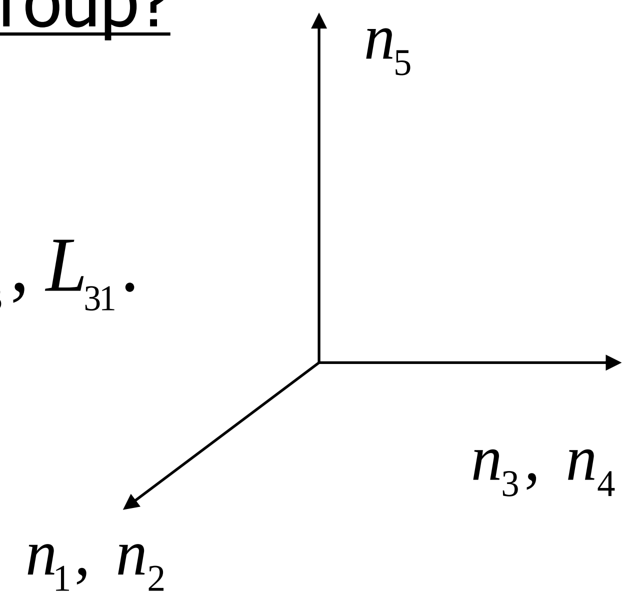
singlet: $$\eta^{+}(i) = \sum_{\alpha\beta} \langle 00 | \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta\rangle c^{+}_{\alpha}(i) c^{+}_{\beta}(i)$$

quintet: $$\chi^{+}_{a}(i) = \sum_{\alpha\beta} \langle 2a | \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta\rangle c^{+}_{\alpha}(i) c^{+}_{\beta}(i)$$




- For arbitrary values of t, μ, $U_0$, $U_2$ and lattice geometry, there is an **exact** Sp(4), or SO(5) symmetry.

# What is Sp(4)(SO(5)) group?




- SU(2) (SO(3)) group.

  3-vector: x, y, z; 3-generator: $L_{12}, L_{23}, L_{31}.$

  2-spinor: $|\uparrow\rangle, \quad |\downarrow\rangle$

- Sp(4)(SO(5)) group.

  5-vector: $n_1, n_2, n_3, n_4, n_5$

  **10-generator**: $L_{ab} \ (1 \le a < b \le 5)$

  4-spinor: $\Uparrow \left|\frac{3}{2}\right\rangle \ \uparrow \left|\frac{1}{2}\right\rangle \ \downarrow \left|-\frac{1}{2}\right\rangle \ \Downarrow \left|-\frac{3}{2}\right\rangle$

- We will see what quantities correspond to these 5-vector and 10-generator.

# spin-3/2 algebra $\psi_\alpha^+ M_{\alpha\beta}\psi_\beta$

- Total degrees of freedom: $4^2=16=1+3+5+7$.

1 density operator and 3 spin operators are far from complete.

| | rank: | | |
|---|---|---|---|
| | 0 | $1,$ | |
| | 1 | $F_x, F_y, F_z$ | |
| $M_{\alpha\beta}$ | 2 | $\xi^a_{ij} F_i F_j \ (a=1\sim 5):$ ↔ | $F_x^2 - F_y^2, F_z^2 - \frac{5}{4},$ $\{F_x, F_y\}, \{F_y, F_z\}, \{F_z, F_x\}$ |
| | 3 | $\xi^a_{ijk} F_i F_j F_k \ (a=1\sim 7)$ | |

- **Spin-quadrupole matrices** (rank-2 tensors) form five-$\Gamma$ matrices (SO(5) vector) --- the same $\Gamma$-matrices in Dirac equation.

$$\Gamma^a = \xi^a_{ij} F_i F_j, \quad \{\Gamma^a, \Gamma^b\} = 2\delta_{ab}, \quad (1 \le a, b \le 5)$$

# Hidden conserved quantities: **spin-octupoles**

- Both $F_{x,y,z}$ and $\xi^a_{ijk} F_i F_j F_k$ commute with Hamiltonian → 10 SO(5) generators: 10=3+7.

- **7 spin-octupole operators** are the hidden conserved quantities.

$$\Gamma^{ab} = \tfrac{i}{2}[\Gamma^a, \Gamma^b] \ (1 \le a < b \le 5)$$

- **SO(5): 1 scalar + 5 vectors + 10 generators = 16**

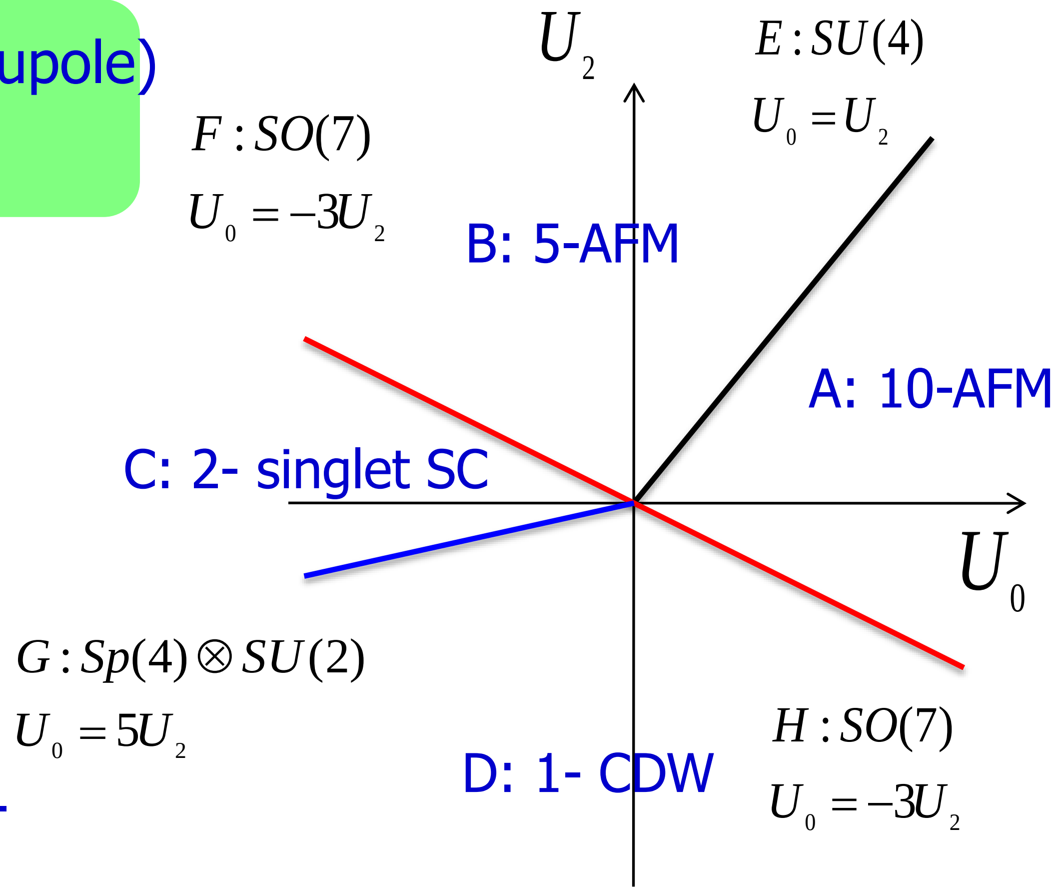
| | | Time Reversal |
|---|---|---|
| 1 density: | $n = \psi^+ \psi;$ | even |
| 5 spin-quadrupole: | $n_a = \frac{1}{2} \psi^+ \Gamma^a \psi;$ | even |
| 3 spins + 7 spin-octupole: | $L_{ab} = \frac{1}{2} \psi^+ \Gamma^{ab} \psi;$ | odd |

# Unify AFM, SC, CDW with **exact** symmetries (half-filled, bipartite lattice)

- SO(7) : AFM (5-spin quadrupole) + SC (singlet).

- Pseudo-spin SU(2): CDW + SC (singlet).
Generalization of Yang's $\eta$-pairing.

- Large symmetry manifold-- the adjoint rep. of SO(7). AFM(10-spin+spin octupole) +SC (10-quintet)+ CDW.



# Sign-problem free QMC algorithm away from half-filling

- An equivalent formulation:

$$H = \sum_{\langle ij\rangle,\sigma} -t\{c^{+}_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_{i} c^{+}_{i,\alpha} c_{i,\alpha} - \sum_{i,1\le a\le 5} \{V(n(i)-2)^2 + W n_a^2(i)\}$$

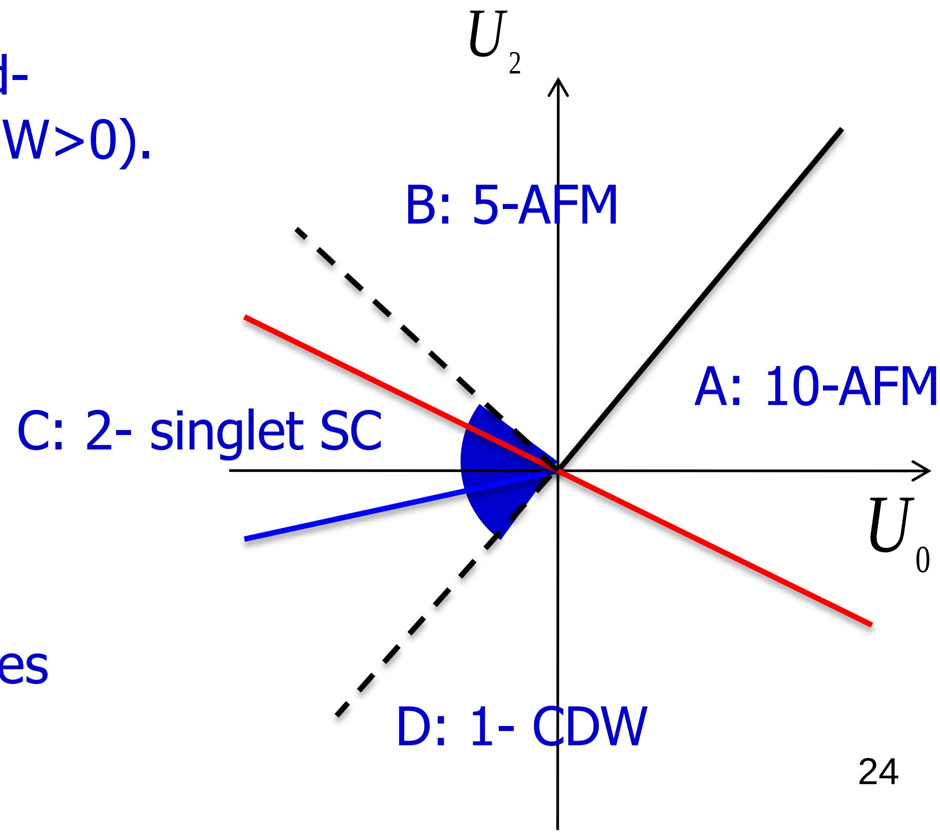
$$V = -\frac{3U_0 + 5U_2}{16}, \quad W = \frac{U_2 - U_0}{4}$$

- Time-reversal invariant Hubbard-Stratonovich decomposition at (V,W>0).

- Fermion determinant remains positive-definite at any filling.

$$U_0 < U_2 < -\frac{3}{5} U_0$$

- Sign problem free region includes Superconductivity, CDW, AFM.

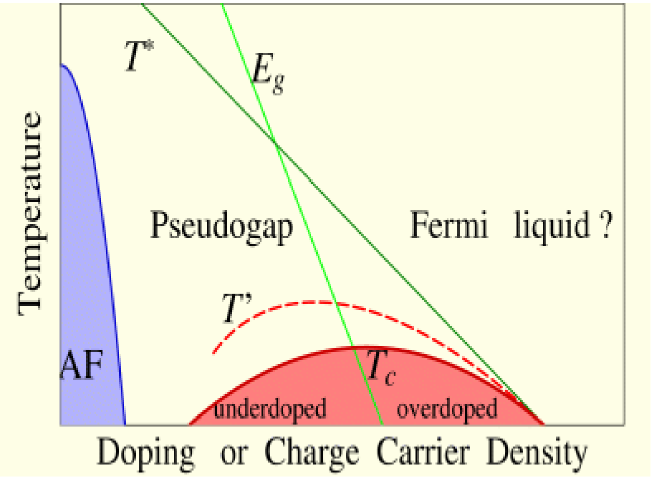


# "Grand-unifications" – elegancy and power of the group theory

- Pseudo-spin SO(3=2+1) unifies SC (singlet) +CDW – C. N. Yang, S. C. Zhang.

- Approx. SO(5=2+3) symm. unifies SC (d-wave singlet) + AFM – S. C. Zhang, E. Demler, et al.

41mev neutron resonance mode in the high Tc SC state: pseudo-Goldstone mode ($\pi$ -mode)




- Exact SO(7=2+5) symm. unifies SC + AFM (5-spin quadrupole).

$$[\chi_a^+, \Delta] = AF_{a,qd} \qquad [H, \chi_a^\pm] = \mp(\mu - \mu_0)\chi_a^\pm$$

5-$\chi$ models: rotate SC←→AF.

$$H(\chi_a^+ | G_{SC}\rangle) = [E_G + (\mu - \mu_0)](\chi_a^+ | G_{SC}\rangle)$$

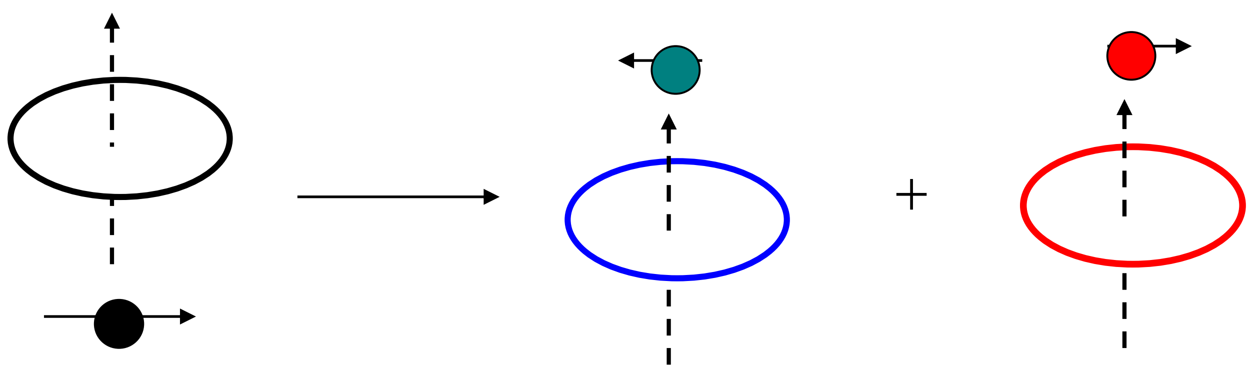
Analogy to the $\pi$-modes in high Tc.

# Non-abelian statistics – Alice vortex loop/particle (SO(4) Cheshire charge)

- Quintet pairing (S=2) → half-quantum vortex loop carrying spin quantum number.

$$|init\rangle = \left|\tfrac{3}{2}\right\rangle_p \otimes |\text{zero charge}\rangle_{vort} \longrightarrow$$

$$|final\rangle = \left|\tfrac{1}{2}\right\rangle_p \otimes |S_z = 1\rangle_{vort} - \left|\tfrac{-1}{2}\right\rangle_p \otimes |S_z = 2\rangle_{vort}$$



$$|00;00\rangle_{vt} \otimes \left|\tfrac{1}{2}\tfrac{1}{2};00\right\rangle_{qp}, \qquad \left|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{-1}{2}\right\rangle_{vt} \otimes \left|00;\frac{1}{2}\frac{1}{2}\right\rangle_{qp} - \left|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}\right\rangle_{vt} \otimes \left|00;\frac{1}{2}\frac{-1}{2}\right\rangle_{qp}.$$



C. Wu, J. P. Hu, and S. C. Zhang, cond-mat/0512602.

# More details

**Brief Review**

Modern Physics Letters B, Vol. 20, No. 27 (2006) 1707–1738


World Scientific
www.worldscientific.com

## HIDDEN SYMMETRY AND QUANTUM PHASES IN SPIN-3/2 COLD ATOMIC SYSTEMS

CONGJUN WU

*Kavli Institute for Theoretical Physics, University of California,*
*Santa Barbara, CA 93106, USA*
*wucj@kitp.ucsb.edu*

Received 31 August 2006

# Outline

- **Introduction: what is large?** large N v.s. large S

- Generic Sp(4) symmetry (spin-$\frac{3}{2}$).

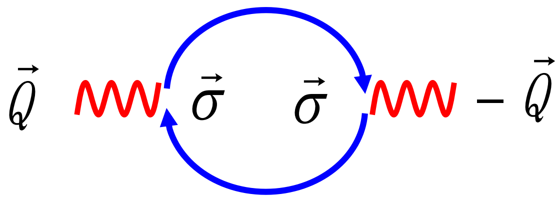
  Unification of antiferromagnetism, superconductivity, and charge-density-wave.
  http://online.kitp.ucsb.edu/online/coldatoms07/wu2/

- Slater v.s. Mott: quantum phase transition at SU(6) -- QMC

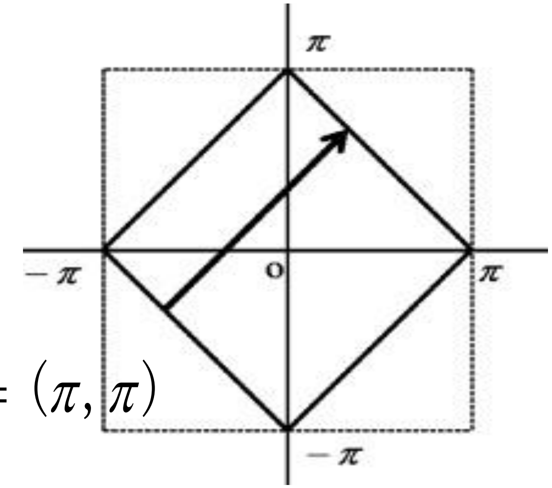
  Interplay between charge and spin degrees of freedom

# SU(2): Slater V. S. Mott (half-filling)

- Fermi surface nesting (small $U/t$) : divergent AFM susceptibility; strong charge fluctuations.



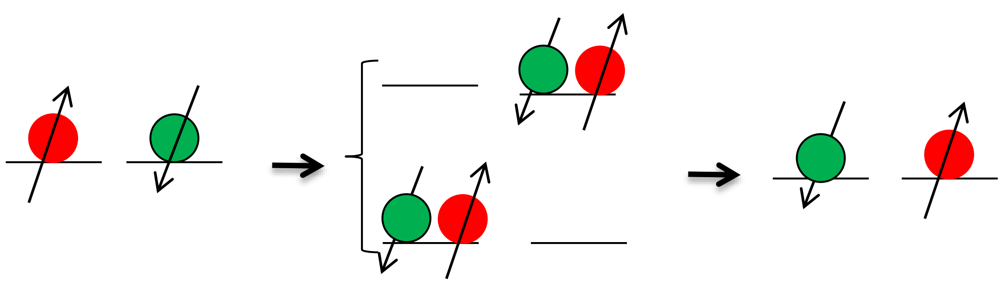

$$m \propto t e^{-\sqrt{\frac{t}{U}}}$$

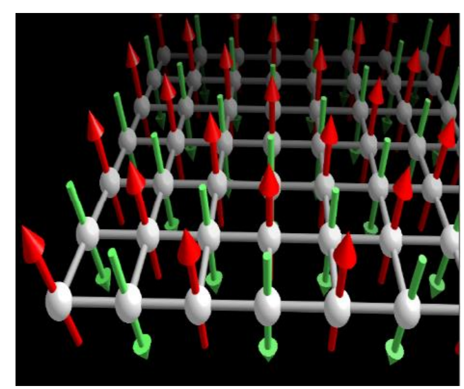



$$\vec{Q} = (\pi, \pi)$$

- Local moments (Large $U/t$) : charge fluctuation suppressed; AFM super-exchange.

$$H = J \sum_i (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4})$$





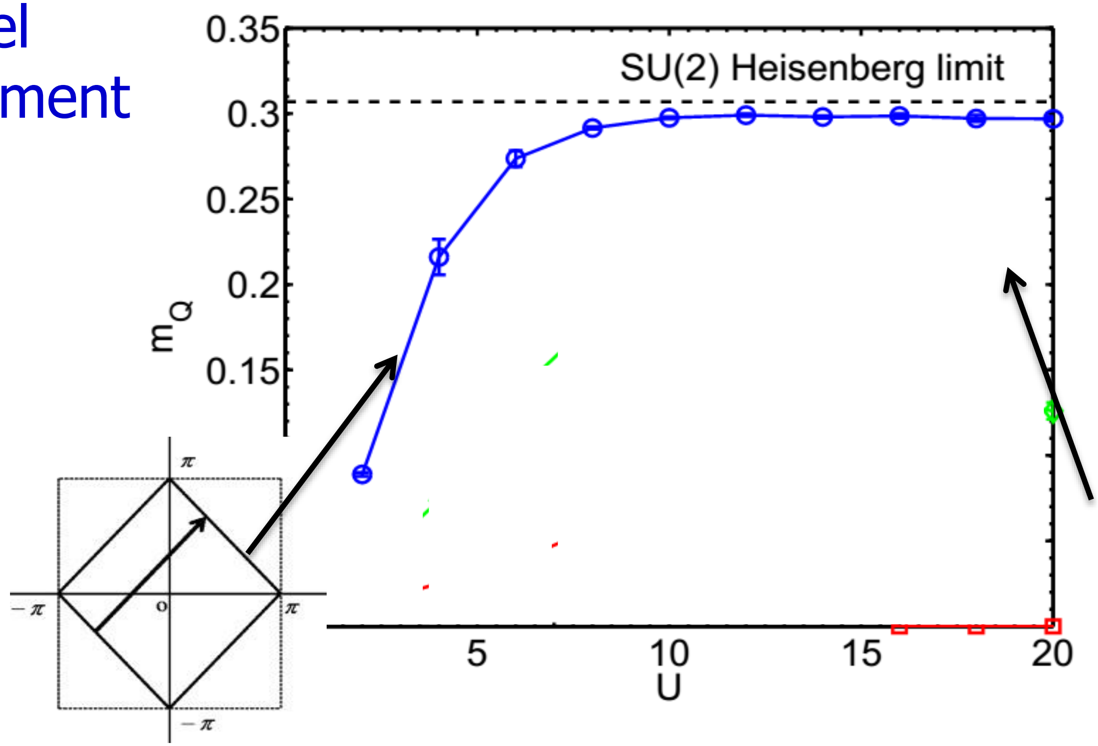
# SU(2): no phase transition

- SU(2): smooth cross-over from Slater to Mott region.

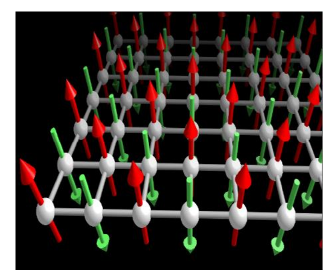
determinant QMC: J. Hirsch, 1985.

D. Wang, Y. Li, Z. Cai, Z. Zhou, Y. Wang, C. Wu, Phys. Rev. Lett. 112, 156403 (2014).

Neel moment




Projector determinant QMC + pinning field.

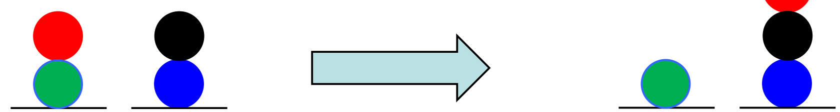


# Half-filled SU(2N) Hubbard model (local moment limit)

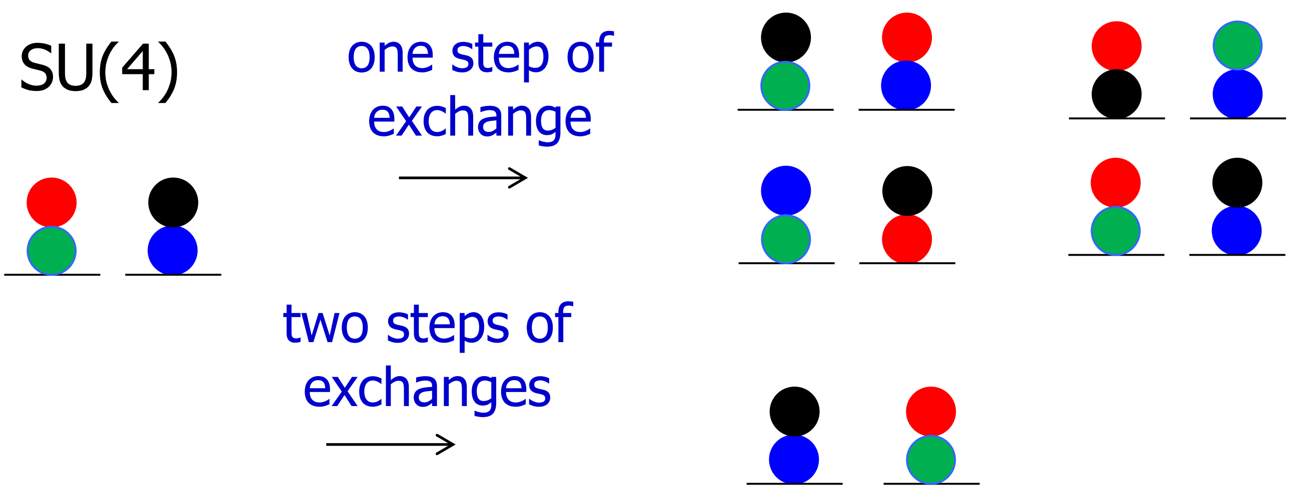
$$H = -t \sum_{\langle ij\rangle,\sigma=1}^{2N} \{c_{i,\sigma}^{+} c_{j,\sigma} + h.c.\} + \frac{U}{2}\sum_i (n_i - N)^2 \qquad n_i = \sum_{\sigma=1}^{2N} n_{i,\sigma}$$

- SU(4) as an example.
In the atomic limit, t=0.

$\Delta E = U$



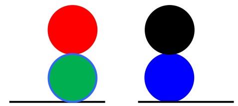
- Turning on t, number of super-exchange processes scales as $N^2$.



# Enhancement of spin fluctuations

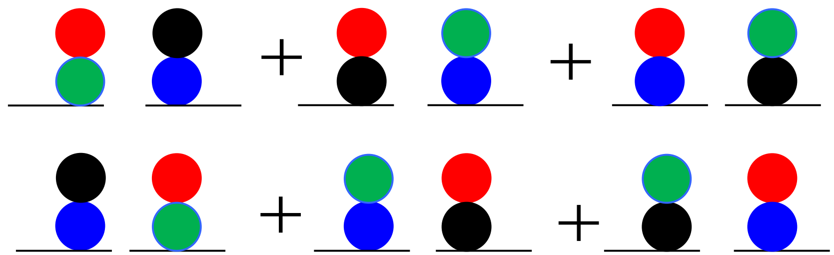
- As increasing 2N, the Neel states become unfavorable.

$$\Delta E = -2zN\frac{t^2}{U}$$



classic-Neel

$$\Delta E = -2N(N+1)\frac{t^2}{U}$$



bond SU(2N) singlet

- Bond dimer state consists of $\binom{2N}{N}$ resonating configurations.

- As N > z (coordination number), valence bond dimerization is favored (Sachdev + Read).

# Projector QMC with the pinning field

- Usual methods to identify long-range-order in simulations:

1) 2-point correlation function: $\lim_{r\to\infty} < S\left(\frac{L}{2}\right) S(0) > \neq 0$

2) Structure factor: $\frac{1}{L^2} \sum < S(r) S(0) > e^{iQr} \neq 0$

Square of order parameter

- The pinning field method (sensitive to weak ordering):

add external field at central sites to explicitly break the symmetry

$$H_{pin,\, n} = h\{m_{i_0} - m_{j_0}\}$$

$$\mathrm{m_Q}(L) = \frac{1}{L^d} \sum \langle S(r) \rangle e^{iQr} \xrightarrow{L \to \infty}$$

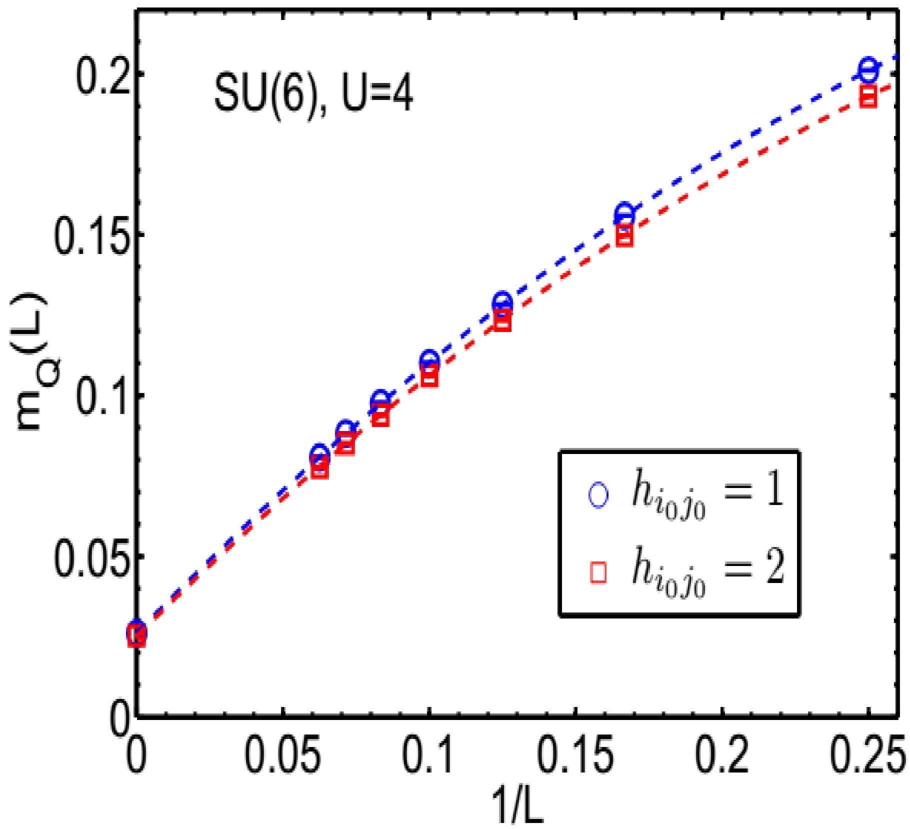
Order parameter

S. R. White and A. L. Chernyshev, PRL (2007); F. F. Assaad and I. F. Herbut, PRX (2013)

# QMC with pinning field: sensitive

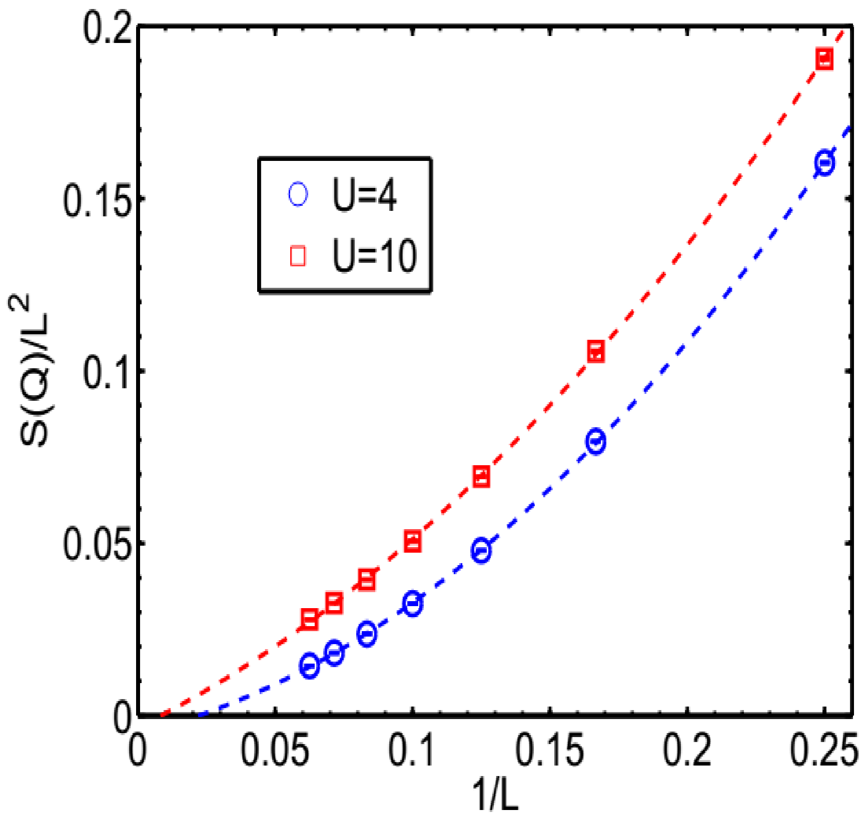
## Pinning field

$$m_Q(L) = \frac{1}{L^d} \sum \langle S(r) \rangle e^{iQr}$$




## Structure factor

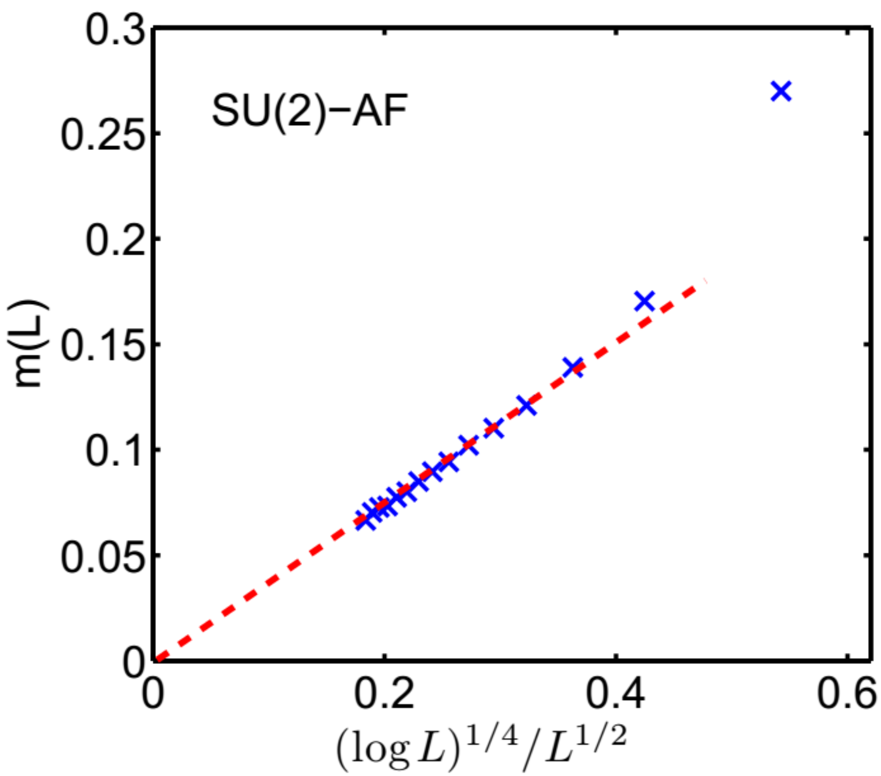
$$S_Q(L) = \sum \langle S(r) S(0) \rangle e^{iQr}$$



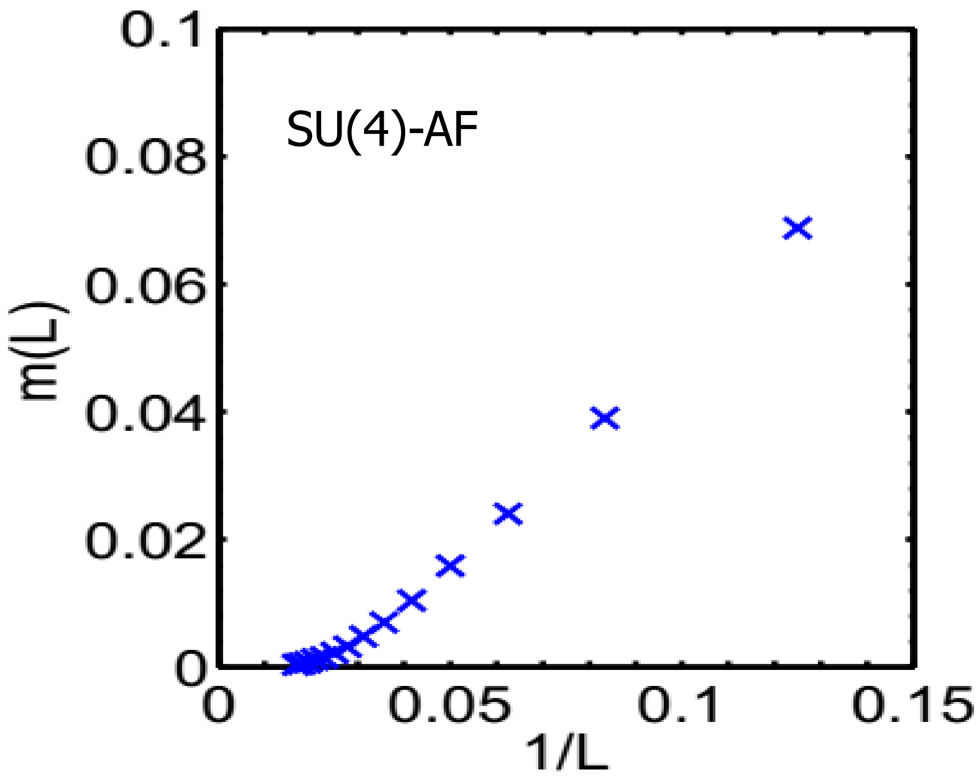
# QMC with pinning field: NOT over-sensitive

- 1D Hubbard model:

SU(2): critical behavior

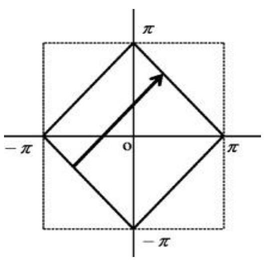



SU(4): no Neel order

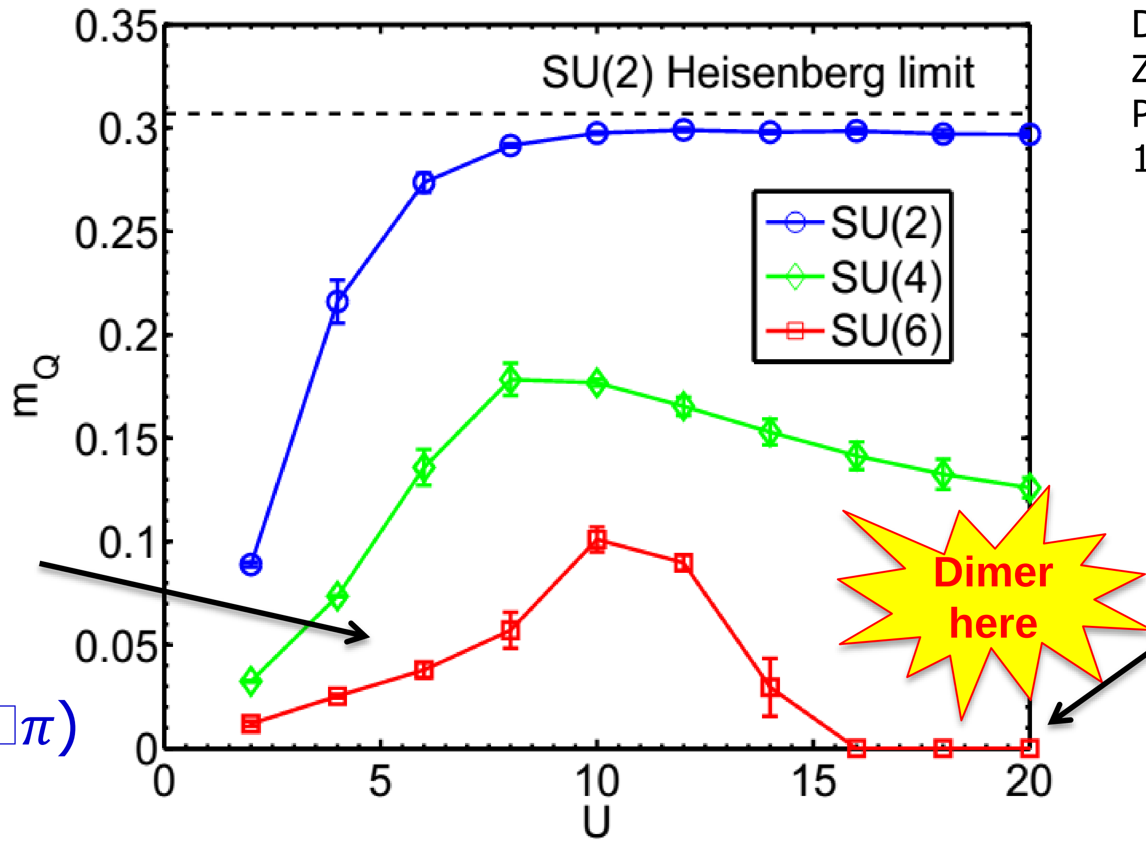


# SU(6): Slater and Mott are different phases

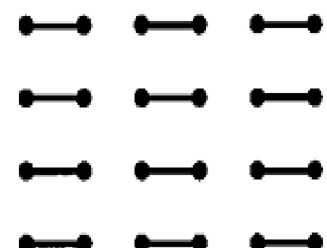
- SU(4) and SU(6): non-monotonic behavior of Neel moment.
- Complete suppression of AFM for SU(6).




AFM $Q=(\pi,\pi)$




D. Wang, Y. Li, Z. Cai, Z. Zhou, Y. Wang, C. Wu, Phys. Rev. Lett. 112, 156403 (2014).



dimer $Q=(\pi,0)$

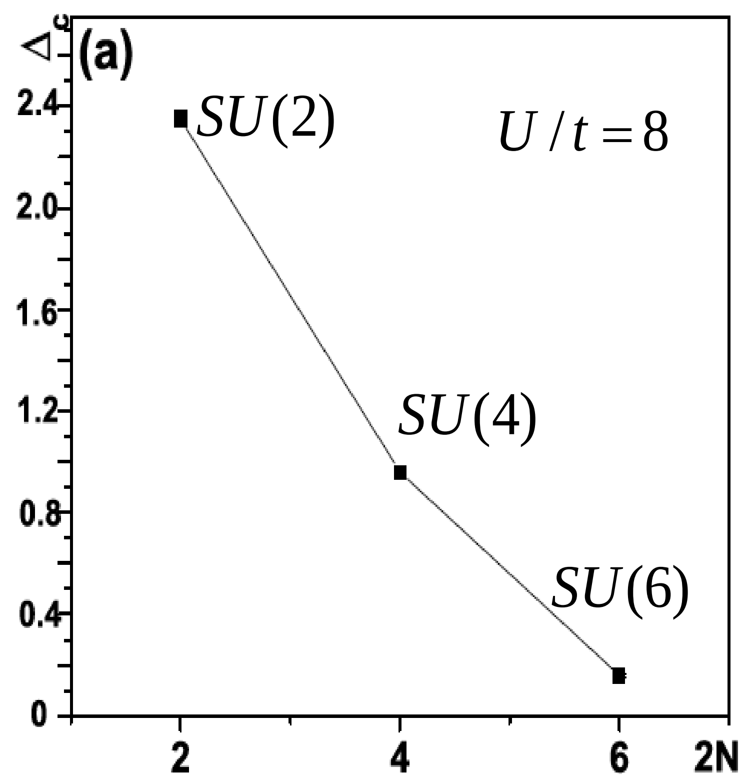
# Mott gap: short-range charge fluctuations

- Single particle gap extracted from Green's function.

$$G(i,i,\tau) = \langle G \mid c_\alpha^+(i,\tau) c_\alpha(i,0) \mid G \rangle$$
$$\rightarrow e^{-\Delta_c \tau}$$

- Mott insulating states do not mean that charge does not move! Charge localization length.

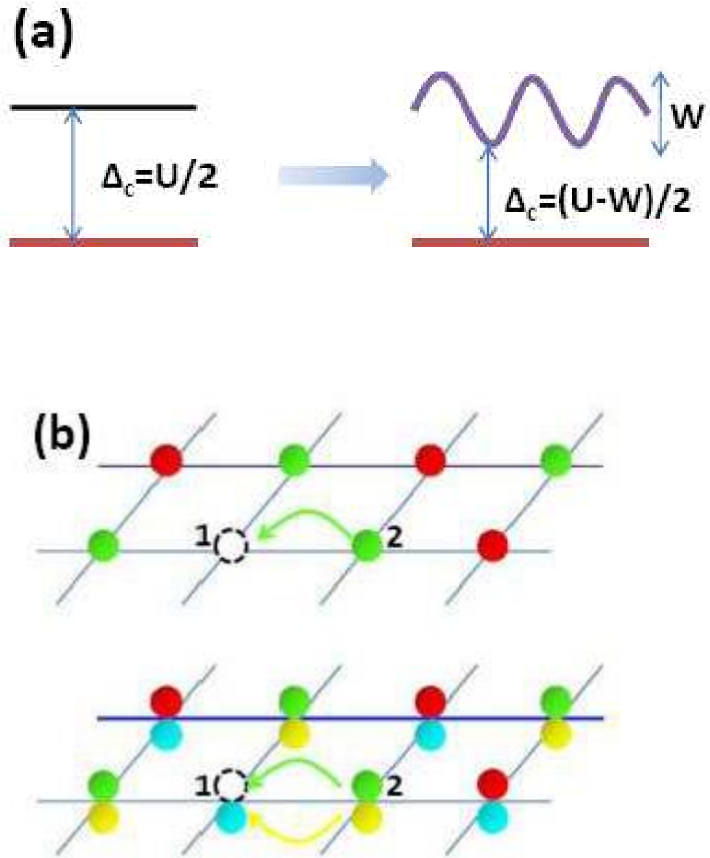
$$\xi_c / a_0 \approx t / \Delta_c$$




- Enhancing charge fluctuations as N increases! It is NOT legitimate to neglect charge degree of freedom.

# Estimation of the single particle gap v.s N (large U)

- Charge gap decreases due to the enhanced number of hopping processes of charge excitations.

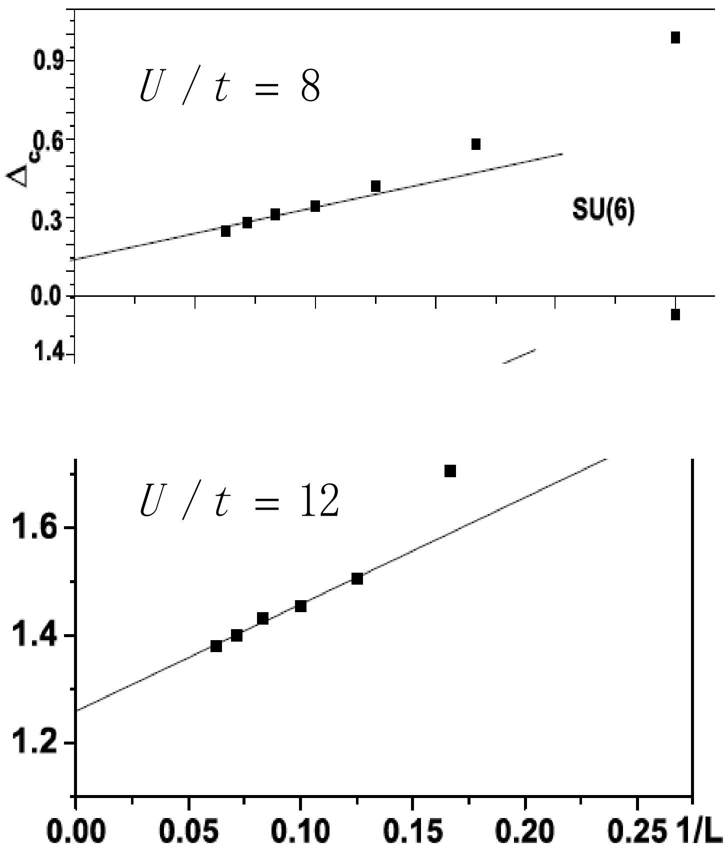



$SU(2)$

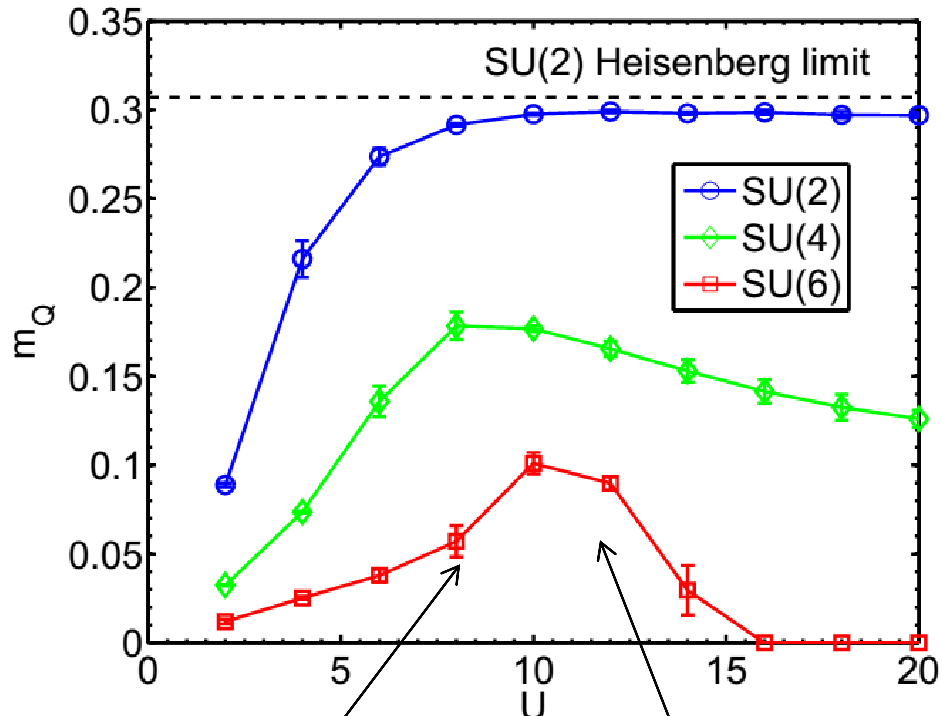
$SU(4)$

$$W \propto Nt$$

# Rapid increase of Mott gap around U~10 (SU(6))



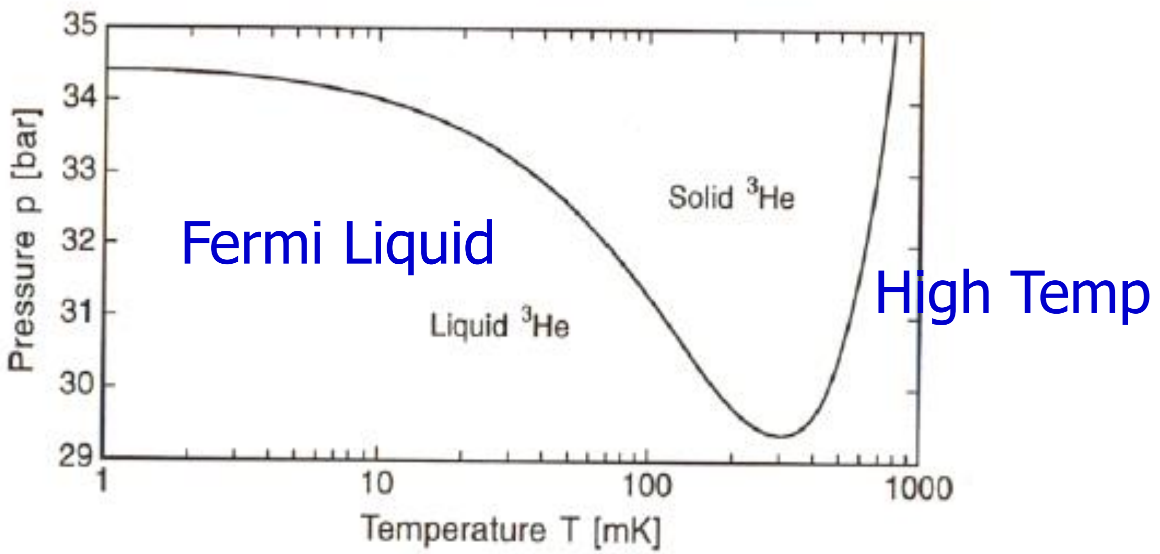

Signature of Mottness




$\Delta_c / t \approx 0.2$ $\Delta_c / t \approx 1.26$

# Thermodynamics: Pomeranchuk effect

- In Mott-insulators, all the sites contribute to entropy through spin configurations, while in Fermi liquids, only fermions close to Fermi surfaces contribute.




- Pomeranchuk effect is more efficient in large spin systems due to the enhanced entropy capability.

S. Taie, arXiv 1208.4883; K. R. Hazzard, et al PRA 2012, Z. Cai et al, PRL, 2013.

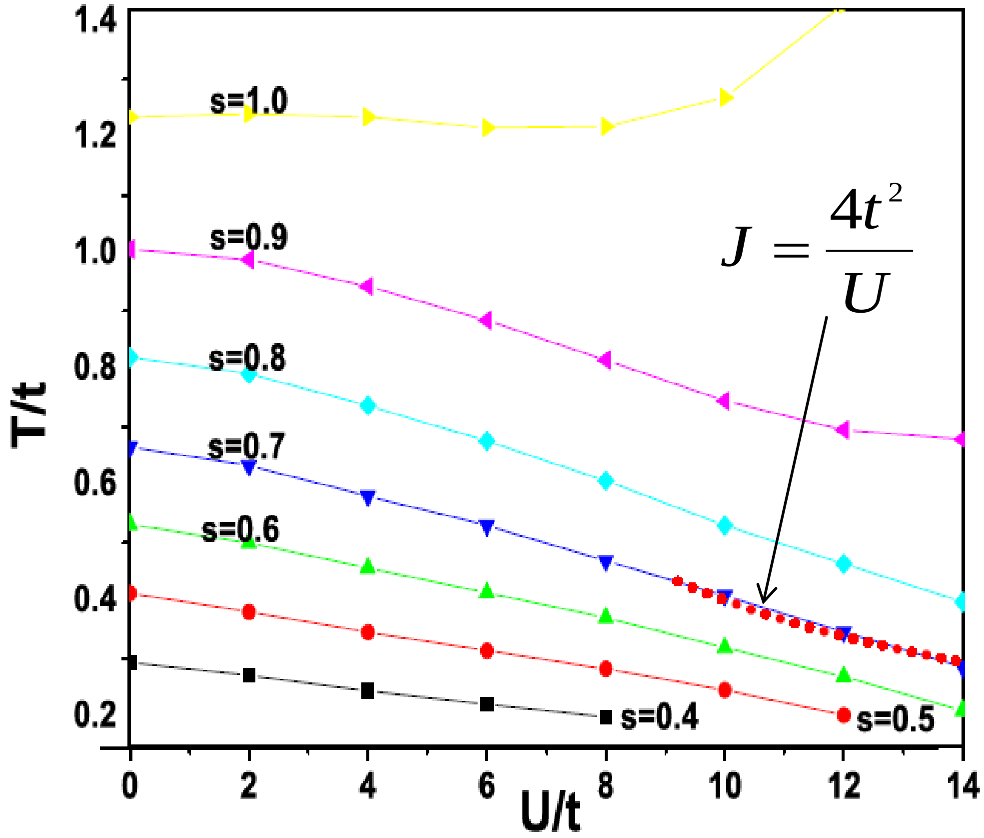
# Pomeranchuk effect (SU(6), half-filling)

- Iso-entropy curve (three-particle per site).

$$S_{su(2N)} = S/(NL^2)$$

$$\frac{S_{su(2N)}(T)}{k_B} = \ln 4 + \frac{E(T)}{T} - \int_T^{\infty} dT' \frac{E(T')}{T'^2},$$

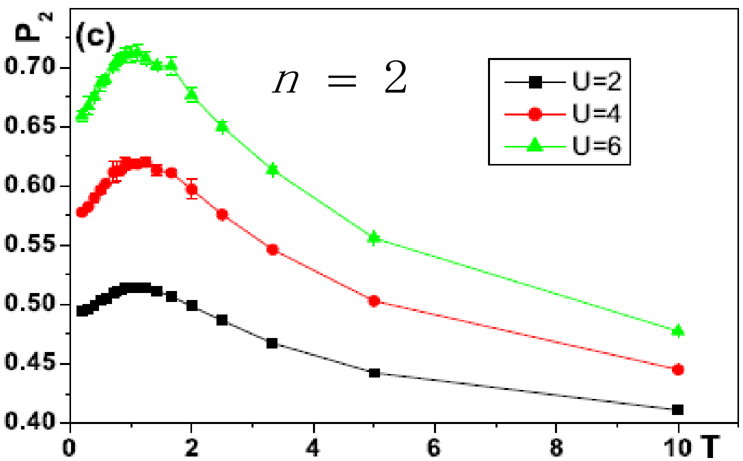
- As entropy per particle s<0.7, increasing U can cool the system below the anti-ferro energy scale J.




Sample size $10 \times 10$

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, PRL 2013.

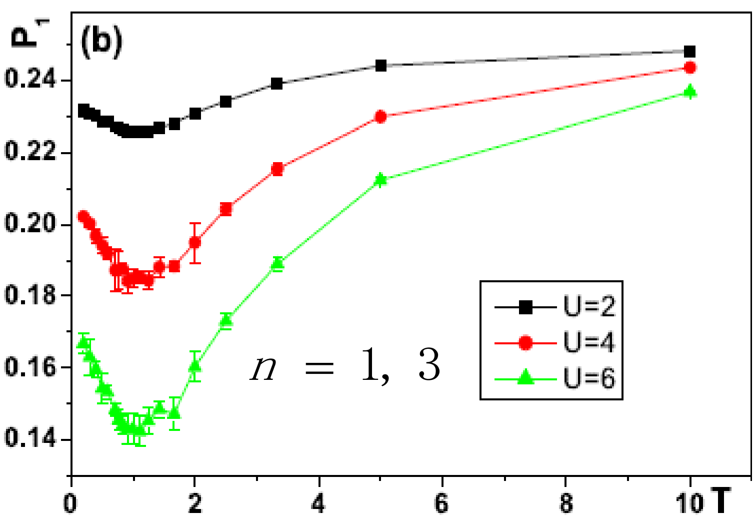
# Probability of onsite occupation (SU(4))

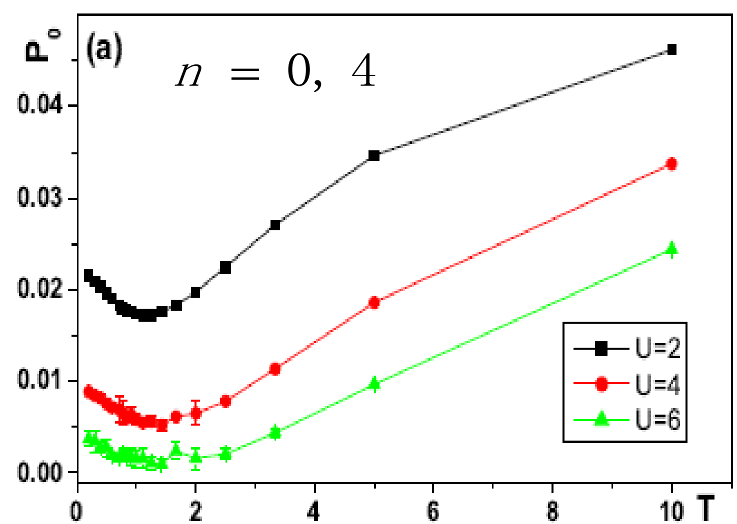



$$P(0) = \prod_{\alpha=1}^{4}(1 - n_i^\alpha);$$

$$P(1) = \sum_{\alpha=1}^{4} n_i^\alpha \prod_{\beta\neq\alpha}(1 - n_i^\beta);$$

$$P(2) = \sum_{\alpha\neq\beta} n_i^\alpha n_i^\beta \prod_{\gamma\neq\alpha\,\beta}(1 - n_i^\gamma).$$



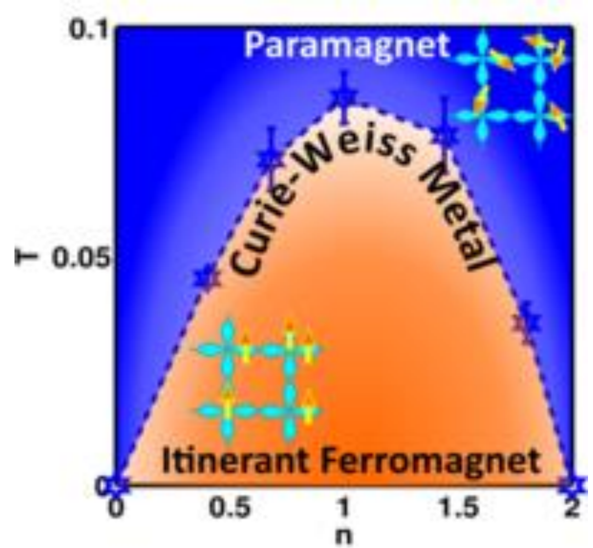
# Digression: itinerant FM based on the Hubbard model

• Fails of the Stoner mechanism: exchange included but correlation neglected!

• A sufficient condition itinerant FM – a rigorous result as a reference point .

Hund's rule + quasi 1D band + strong correlation → itinerant FM

Y. Li, E. Lieb, C. Wu, PRL 112, 217201 (2014).




• Non-perturbative QMC study on Cure-Weiss metals and magnetic transitions.

• Cold atom p-orbital systems and $SrTiO_3/LaAlO_3$ interface.

S. Xu, Y. Li, C. Wu, Phys. Rev. X 5, 021032, (2015).

# Conclusion

- **Large-spin cold fermions are quantum-like NOT classical**!

- Elegancy of unification (group theory based on Sp(4)):

  AFM, SC and CDW phases/ Non-abelian Alice/Cheshire physics

- SU(6) Mott-ness: competition between Fermi surface (Slater) and local moments (Mott).

  Quantum phase transitions inside the insulating regime.

- Pomeranchuk effect of 2D SU(6) Hubbard model.