Unconventional magnetism and spontaneous spin-orbit ordering

Congjun Wu (Univ. California, San Diego)

Ref. 1) C. Wu and S. C. Zhang, PRL 93, 36403 (2004);
3) Y. Li, C. Wu, PRB 85, 205126 (2012).

Jan 18, 2017, UCSD
Collaborators

- S. C. Zhang, Stanford.
- E. Fradkin, UIUC.
- D. Arovas, UCSD
- K. Sun, UIUC (now at U. Michigan)
- W. C. Lee, UCSD (now at Binghamton, SUNY)
- Y. Li, UCSD (now at Johns Hopkins)
- C Xu and S. L. Xu, UCSD

Introduction

unconventional magnetism

unconventional superconductivity

unconventional symmetries

nematic electron liquid with spin

spin-orbit coupling
**Itinerant FM: Quantum origin!**

- Q: How does spin-independent interaction induce spin polarization?
- Electrons with parallel spins avoid each other to reduce repulsion.

**E. C. Stoner**

- **Stoner criterion:**
  \[ UN_0 > 1 \]
  U: interaction strength

\[ E_{\uparrow\uparrow} < E_{\uparrow\downarrow} \]
Itinerant ferromagnetism: \textit{s-wave}

- Spin rotational symmetry is broken.

- Orbital rotational symmetry is \textbf{NOT} broken: spin polarizes along \textbf{a fixed direction}.

\textit{cf.} Conventional superconductivity.

\textit{s-wave}: gap function invariant over the Fermi surface.
**cf.** Unconventional superconductivity

- High partial wave pairing symmetries (e.g. $p$, $d$-wave ...).

- $d$-wave: high $T_c$ cuprates.

- $p$-wave: $\text{Sr}_2\text{RuO}_4$, $^3\text{He}$-A and B.

New states of matter: **unconventional magnetism**!

- High partial wave channel magnetism (e.g. \( p, d\)-wave...).

- Multi-polar spin distribution over the Fermi surface.


Spin flips the sign as \( \vec{k} \rightarrow -\vec{k} \).
Introduction: electron spin liquid-crystal

- unconventional superconductivity
- unconventional magnetism
- anisotropic electron liquid
- Spin-orbit coupling
Anisotropy: liquid crystalline order

- Classic liquid crystal.

  Nematic phase: rotational anisotropic but translational invariant.

- Quantum version of liquid crystal: nematic electron liquid.

Nematic electron liquid in 2D GaAs/AlGaAs at high B fields


M. P. Lilly et al., PRL 82, 394 (1999)
Nematic electron liquid in $\text{Sr}_3\text{Ru}_2\text{O}_7$ at high B fields

- Quasi-2D system; resistivity \textit{anisotropy} at $\sim 8$ Tesla.
- Fermi surface nematic distortions.

**Anisotropic unconventional magnetism: spin liquid-crystal phases!**

- \( p \)-wave distortion of the Fermi surface.

- Spin dipole moment in momentum space (not in coordinate space).

\[ \vec{n}_1 = \sum_k \vec{s}_k \cos \theta_k \neq 0 \]

- Both orbital and spin rotational symmetries are broken.

---

**Spin-split state** by J. E. Hirsch, PRB 41, 6820 (1990); PRB 41, 6828 (1990).

Introduction: dynamic generation of spin-orbit coupling

unconventional magnetism

unconventional superconductivity

anisotropic electron liquid

Spin-orbit coupling
Unconventional magnetism: dynamic generation of spin-orbit (SO) coupling!

• Conventional wisdom:

  A single-body effect from the Dirac equation

• New mechanism (many-body collective effect):

  Generate SO coupling through unconventional magnetic phase transitions.

• Advantages: tunable SO coupling by varying temperatures; new types of SO coupling.
The isotropic $\rho$-wave magnetic phase

- Helicity $\vec{\sigma} \cdot \vec{k}$ is a good quantum number.

- No net spin-moment; spin dipole moment in momentum space.

$$\vec{n}_1 = \sum_k \vec{s}_k \cos \theta_k, \quad \vec{n}_2 = \sum_k \vec{s}_k \sin \theta_k$$

- Isotropic phase with SO coupling.

$$H_{MF} = H_0 + \vec{n} \sum_k \psi^+_\alpha \vec{\sigma}_{\alpha\beta} \cdot \vec{k} \psi_\beta$$

$$\vec{n} = |\vec{n}_1| = |\vec{n}_2|$$

C. Wu et al., PRL 93, 36403 (2004);
C. Wu et al., PRB PRB.75, 115103 (2007).
The subtle symmetry breaking pattern

- $\vec{J}$ is conserved, but $\vec{L}, \vec{S}$ are not separately conserved.

- Independent orbital and spin rotational symmetries.

- Relative spin-orbit symmetry breaking.

$\vec{J} = \vec{L} + \vec{S}$

Leggett, Rev. Mod. Phys 47, 331 (1975)
Summary of the introduction

unconventional superconductivity

unconventional magnetism

electron liquid crystal with spin

spin-orbit coupling
• **Introduction.**

• **Mechanism for unconventional magnetic phase transitions.**
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.

• **Spin-orbit coupled Fermi liquid theory – magnetic dipolar.**

• **Possible directions of experimental realization and detection methods.**
Landau Fermi liquid (FL) theory

- The existence of Fermi surface.
- Electrons close to Fermi surface are important.

- Interaction functions:

\[
\begin{align*}
 f_{\alpha\beta,\gamma\delta}(\hat{p}_1, \hat{p}_2) &= f^s(\hat{p}_1, \hat{p}_2) \\
 &+ f^a(\hat{p}_1, \hat{p}_2) \tilde{\sigma}_{\alpha\beta} \cdot \tilde{\sigma}_{\gamma\delta}
\end{align*}
\]

- Landau parameter in the \( l \)-th partial wave channel:

\[
F_{l}^{s,a} = N_{0} f_{l}^{s,a} \\
N_{0} : \text{DOS}
\]
Pomeranchuk instability

- Fermi surface: elastic membrane.
- Stability:
  \[
  \Delta E_K \propto (\delta n_l^{s,a})^2 \\
  \Delta E_{\text{int}} \propto \frac{F_l^{s,a}}{2l + 1} (\delta n_l^{s,a})^2
  \]
- Surface tension vanishes at:
  \[
  F_l^{s,a} < -(2l + 1)
  \]
- Ferromagnetism: the \( F_0^a \) channel.
- Nematic electron liquid: the \( F_2^s \) channel.
Unconventional magnetism: Pomeranchuk instability in the spin channel

\[ F_1^a \]

\[ k_{f1}, k_{f2} \]

\[ \vec{s}, \vec{S} \]

\( \beta \) – phase

\( \alpha \) – phase

• An analogy to superfluid \(^3\)He-B (isotropic) and A (anisotropic) phases.
**cf. Superfluid $^3\text{He-B, A phases}$**

- $\rho$-wave triplet Cooper pairing.

- $^3\text{He-B (isotropic) phase.}$

- $^3\text{He-A (anisotropic) phase.}$

A. J. Leggett, Rev. Mod. Phys 47, 331 (1975)
The order parameters: the 2D $p$-wave channel

• $F_1^a$: Spin currents flowing along x and y-directions, or spin-dipole moments in momentum space.

\[ \vec{n}_1 = \sum_k \psi_k^+ \vec{\sigma} \psi_k \cos \theta_k \]
\[ \vec{n}_2 = \sum_k \psi_k^+ \vec{\sigma} \psi_k \sin \theta_k \]

• cf. Ferromagnetic order (s-wave): \[ \vec{s} = \sum_k \psi_k^+ \vec{\sigma} \psi_k \]

• Arbitrary partial wave channels: spin-multipole moments.

\[ F_l^a : \cos \theta_k \to \cos l \theta_k ; \ \sin \theta_k \to \sin l \theta_k \]
Mean field theory and Ginzburg-Landau free energy

- The simplest non-$s$-wave exchange interaction:

\[
F_1^a = \sum_q f_1^a(\bar{q})\{\vec{n}_1(\bar{q}) \cdot \vec{n}_1(\bar{q}) + \vec{n}_2(\bar{q}) \cdot \vec{n}_2(\bar{q})\}
\]

\[
H_{MF} = \sum_k \psi^+(k)[\varepsilon(k) - \mu - (\vec{n}_1 \cos \theta_k + \vec{n}_2 \sin \theta_k) \cdot \vec{\sigma}]\psi(k)
\]

- Symmetry constraints: rotation (spin, orbital), parity, time-reversal.

\[
F(\vec{n}_1, \vec{n}_2) - F(0) = r(|\vec{n}_1|^2 + |\vec{n}_2|^2) + v_1(|\vec{n}_1|^2 + |\vec{n}_2|^2)^2 + v_2 |\vec{n}_1 \times \vec{n}_2|^2
\]

\[
r = \frac{N_0}{2} \frac{1 + F_1^a / 2}{|F_1^a|} \quad F_1^a < -2 \quad \text{instability!}
\]
\[ \beta \text{ and } \alpha \text{-phases (}\rho\text{-wave)} \]

\[ F^a_1 \]

\[ \nu_2 < 0: \beta - \text{phase} \]
\[ \vec{n}_1 \perp \vec{n}_2 \text{ and } |\vec{n}_1| = |\vec{n}_2| \]

\[ \nu_2 > 0: \alpha - \text{phase} \]
\[ \vec{n}_1 \parallel \vec{n}_2; |\vec{n}_2| / |\vec{n}_1| \text{ arbitrary} \]
The $\beta$-phases: vortices in momentum space

- Perform global spin rotations, $A \rightarrow B \rightarrow C$.

L. Fu’s (PRL2015): gyro            ferroelectric            muti-polar
2D $d$-wave $\alpha$ and $\beta$-phases

$\alpha$-phase

$\beta$-phase: $w=2$

$\beta$-phase: $w=-2$
The $\alpha$-phases: orbital & spin channel Goldstone (GS) modes

- Orbital channel GS mode: FS oscillations (intra-band transition).

\[ L_{FS}^\alpha (\vec{q}, \omega) = N_0 \left\{ \frac{(q \xi)^2}{|F_l^a|} - i \frac{\omega}{2v_f q} (1 + \cos 2\phi_q) \right\} \]

Anisotropic overdamping: The mode is maximally overdamped for $q$ along the $x$-axis, and underdamped along the $y$-axis ($l=1$).

- Spin channel GS mode: “spin dipole” precession (spin flip transition).

\[ \omega_{x\pm iy}^2 = \frac{n^2}{|F_l^a|} (q \xi)^2 \]

Nearly isotropic, underdamped and linear dispersions at small $q$. 
The $\alpha$-phases: neutron spectra

- No *elastic* Bragg peaks.

- $\vec{n}_{1,2}$ can couple with spin dynamically at $T < T_c$ -- coupling between GS modes and spin-waves (spin-flip channel).

\[
\begin{align*}
L &= (\vec{n}_1 \times \partial_t \vec{n}_1 + \vec{n}_2 \times \partial_t \vec{n}_2) \cdot \vec{\mathbb{S}} \\
&\rightarrow \vec{n} \left( S_y \partial_t n_{1x} - S_x \partial_t n_{1y} \right)
\end{align*}
\]

- *In-elastic*: resonance peaks develop at $T < T_c$.

\[
\begin{align*}
\text{Im} \chi_{s,\pm}(\vec{q}, \omega) &\approx N_0 \omega_q^2 \delta(\omega^2 - \omega_q^2)
\end{align*}
\]
The $\beta$-phases: GS modes

- 3 branches of relative spin-orbit modes.

\[ O_z = \frac{1}{\sqrt{2}} (n_2^x - n_1^y); \]
\[ O_x = -n_2^z; \quad O_y = n_1^z; \]

- For $l \geq 2$, these modes are with linear dispersion relations, and underdamped at small $q$.

- Inelastic neutron spectra: GS modes also couple to spin-waves, and induce resonance peaks in both spin-flip and non-flip channels.
• Introduction.

• Mechanism for unconventional magnetic phase transitions.
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.

• Spin-orbit coupled Fermi liquid theory – magnetic dipolar interaction.

• Possible directions of experimental realization and detection methods.
Magnetic dipoles: from classic to quantum

- Ferro-fluid: iron powders in oil.

- In solids, magnetic dipolar interaction $\ll$ Coulomb interaction.

\[
    r_s = \frac{d}{a_B} \quad \quad \quad E_m = \frac{\mu_B^2}{d^3} = \frac{\lambda_{cmp}^2 \, R_y}{a_B^2 \, r_s^3} = \frac{\alpha^2}{r_s^2} \, E_{el} \approx \frac{1.4}{r_s^3} \, meV
\]
Magnetic dipolar Fermi gases

- Itinerant magnetic dipolar system: \(^{(161}\text{Dy}, 163\text{Dy}) 10\mu_B\)

\[ n \approx 4 \times 10^{13} \text{cm}^{-3}, T_F \approx 300 \text{nK} \quad \lambda = \frac{E_d}{E_f} \approx 15\% \]

- SO coupling at the interaction level.

\[ V(\vec{r}) = \frac{(g\mu)^2}{r^3} [\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \hat{r})(\vec{F}_2 \cdot \hat{r})] \]

- SO coupled many-body physics (no Fermi surface splitting):


Spin-orbit (SO) coupled Fermi liquid theory

- Landau functions: SO harmonic partial-wave decomposition.

\[
\frac{N_0}{4\pi} f_{\alpha\alpha',\beta\beta'}(\hat{k}_1, \hat{k}_2) = \sum_{JJ_zLL'} Y_{JJ_z;LS}(\hat{k}_1, \alpha\alpha') \hat{F}_{JJ_z;LS, JJ_z'L'S} Y^{+}_{JJ_z;L'S}(\hat{k}_2, \beta\beta')
\]

- Landau matrices: an eigenvalue < -1 → Pomeranchuk instability

- $J = 1^-(\text{odd parity}), L = S = 1$.

- Transfer SO coupling to the single particle level (Rashba like).

Y. Li, C. Wu, PRB 85, 205126 (2012).
Topological SO zero sound

• SO coupled Fermi surface oscillations.
  \( u_0 \): hedgehog distribution: \( J = 0^- \);
  \( u_1 \): longitudinal ferro: \( J = 1^+ \), and \( u_0 > u_1 \)

\[
\vec{S}(\vec{r}, \vec{k}, t) = (u_0 \hat{k} + u_1 \hat{q}) e^{i(\vec{q} \cdot \vec{r} - \omega t)}
\]

• Underdamped mode
  \( s = \omega/(v_f q) > 1 \): sound velocity > Fermi velocity.

\[
\begin{align*}
 s_{\lambda \ll 1} & \approx 1 + 2e^{-2(1+1/2F_+)} = 1 + 2e^{-2-12/7\pi \lambda}, \\
 s_{\lambda \gg 1} & \approx \frac{F_\times}{3} = \frac{\pi}{3\sqrt{3}} \lambda.
\end{align*}
\]

\[
F_+ = F_{10;01} + F_{00;11} \quad F_\times = \sqrt{F_{10;01}F_{00;11}}
\]
• Introduction.

• Mechanism for unconventional magnetic phase transitions.
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.

• Spin-orbit coupled Fermi liquid theory – magnetic dipolar interaction.

• Possible directions of experimental realization and detection methods.
A natural generalization of ferromagnetism

- The driving force is still exchange interactions, but in **non-s-wave** channels.

<table>
<thead>
<tr>
<th></th>
<th>$s$-wave</th>
<th>$p$-wave</th>
<th>$d$-wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC/SF</td>
<td>Hg, 1911</td>
<td>$^3$He, 1972</td>
<td>high $T_c$, 1986</td>
</tr>
<tr>
<td>magnetism</td>
<td>Fe, ancient time</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

- Optimistically, unconventional magnets are probably not rare.

cf. Antiferromagnetic materials are actually very common in transition metal oxides. But they were not well-studied until neutron-scattering spectroscopy was available.
Search for unconventional magnetism (I)

- **URu$_2$Si$_2$:** hidden order behavior below 17.5 K.

T. T. M. Palstra et al., PRL 55, 2727 (1985);
M. B. Maple et al, PRL 56, 185 (1986)

**helicity order (the $p$-wave $\alpha$-phase);**

Search for unconventional magnetism (II)

- \( \text{Sr}_3\text{Ru}_2\text{O}_7 \) in the external \( B \) field – Orbital-assisted unconventional meta-magnetic state.

\[
f_{\uparrow\uparrow}(\vec{p}_1, \vec{p}_2) = V(q = 0) - \frac{1}{2} [1 + \cos 2\theta_{p_1p_2}] V(p_1 - p_2)
\]

\[
f_{\uparrow\downarrow}(\vec{p}_1, \vec{p}_2) = V(q = 0)
\]

W. C. Lee, C. Wu, PRB 80, 104438 (2009)
Direct Observation of Nodes and Twofold Symmetry in FeSe Superconductor

Can-Li Song,1,2 Yi-Lin Wang,2 Peng Cheng,1 Ye-Ping Jiang,1,2 Wei Li,1 Tong Zhang,1,2 Zhi Li,2 Ke He,2 Lili Wang,2 Jin-Feng Jia,1 Hsiang-Hsuan Hung,3 Congjun Wu,3 Xucun Ma,2* Xi Chen,1* Qi-Kun Xue1,2

• Consistent with orbital ordering between dxz/dyz orbitals.

Detection (I): ARPES

- Angular Resolved Photo Emission Spectroscopy (ARPES).

ARPES in spin-orbit coupling systems (Bi/Ag surface), Ast et al., cond-mat/0509509.

**band-splitting for two spin configurations.**

- $\alpha$ and $\beta$-phases (dynamically generated spin-orbit coupling):

  The band-splitting is proportional to order parameter, thus is temperature and pressure dependent.
Detection (II): neutron scattering and transport

- Elastic neutron scattering: no Bragg peaks;
Inelastic neutron scattering: resonance peaks below $T_c$.

- Transport properties.

$\beta$-phases: Temperate dependent beat pattern in the Shubnikov-de Hass magneto-oscillations of $\rho(B)$.

Detection (III): transport properties

- Spin current induced from charge current (d-wave). Their directions are symmetric about the x-axis.

\[
\begin{pmatrix}
  j_{x,\uparrow} \\
  j_{x,\downarrow} \\
  j_{y,\uparrow} \\
  j_{y,\downarrow}
\end{pmatrix}
\propto
\begin{pmatrix}
  1 & -1 \\
  -1 & 1
\end{pmatrix}
\begin{pmatrix}
  j_{x,\text{charge}} \\
  j_{y,\text{charge}}
\end{pmatrix}
\]
Summary

unconventional superconductivity

unconventional magnetism

α-phase

electron liquid crystal with spin

spin-orbit coupling

β-phase