Novel *p*-wave pairing with ultra-cold electric and magnetic dipolar fermions

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Ref.
1. C. Wu and J. E. Hirsch, PRB 81, 20508 (2010).
2. Yi Li , C. Wu, Scientific Report 2, 392 (2012) (arxiv:1005.0889).
3. Yi Li , C. Wu, PRB 85, 205126 (2012).

Stanford, Nov 30,2012

Outline

• Difference between electric and magnetic dipolar interactions.

• Multi-component electric dipolar fermions: competition between triplet and singlet pairings.

Mixture between p-wave triplet and s+d single parings →Time-reversal symmetry breaking

• Magnetic dipolar fermion systems: p-wave spin triplet with total angular momentum 1 (L=S=J=1)

A new pairing symmetry, not another He3.

• Fermi liquid: a "topological" collective mode of Fermi surface oscillation.

Dipolar interactions

• Electric dipolar interaction is the leading order interaction in charge neutral systems.

• Electrons carry spins, thus magnetic dipolar interaction is actually one part of fundament interactions among electrons.

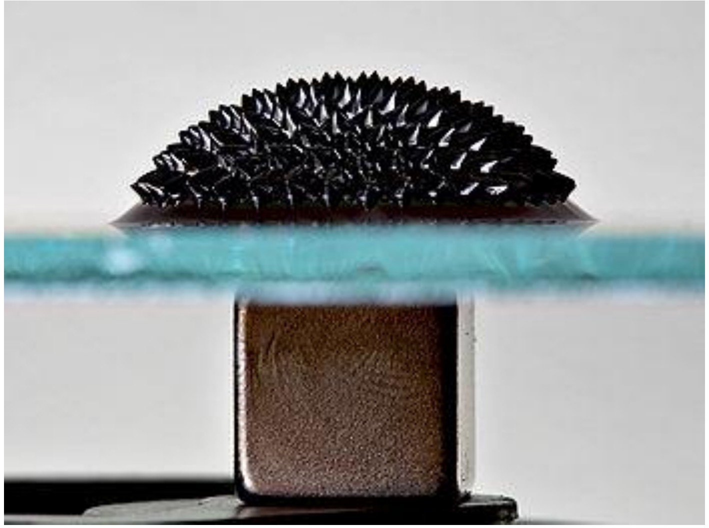
$$p \approx e a_B \quad E_{el} = \frac{p^2}{d^3} = Ry\left(\frac{a_B}{d}\right)^3 \quad E_m = \frac{\mu_B^2}{d^3} = Ry\frac{a_B \lambda_{cmptn}^2}{d^3} = \alpha^2 E_{el}$$

• Unlike Coulomb interaction, dipolar interactions are anisotropic if dipoles are aligned.

• Dipolar interactions are not purely repulsive, not purely attractive → both interesting pairing and zero sound modes.

Classic dipoles

- Ferro-fluid: iron powders in oil.

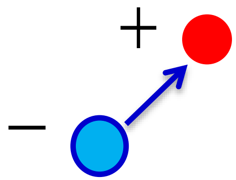


- Quantum dipolar Fermi gases may be easier to handle because most degrees of freedom are blocked by Fermi surfaces.

Difference between electric and magnetic ones

• Electric dipole moments are non-quantized vectors.

not permanent



$$\vec{p} = \int d\vec{r}\,\rho(r)\,\vec{r}$$

• Although at each instant time, there is a dipole-moment, it is NOT an angular momentum eigen-state, and thus averages to zero.

• So far, no permanent electric-dipole moments are found.

• Need external E fields to induce electric dipole moments.

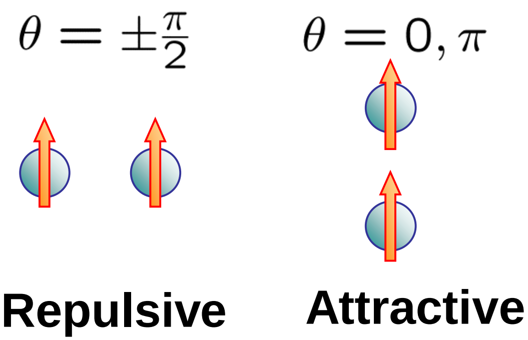
Electric dipolar gas: $^{40}K^{87}Rb$ polar molecules [Ni et. al, 2008, etc …]
M. A. Baranov, M. Dalmonte, G. Pupillo, P. Zoller, arxiv:1207.1914.

Anisotropic interaction between electric dipoles

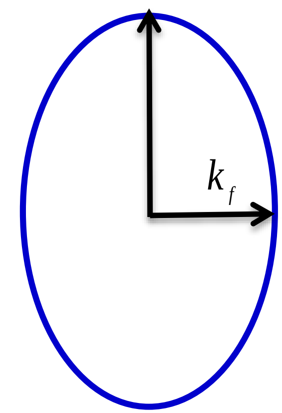
- A nice anisotropy: second Legendre polynomial; quasi-long range.

$$V(\vec{r}) = -\frac{d^2}{r^3} P_2(\cos\theta)$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$



- **Anisotropic many-body physics:** Fermi liquid theory and p-wave Cooper pairing.



anisotropic Fermi surface distortion; anisotropic collective sound modes etc

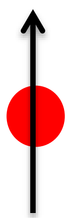
Baranov et al, PRA 66, 013606 (2002); PRL 92, 250403 (2004).

T. Miyakawa, H. Pu, S. Yi, PRA 77, 2008. Chan, Wu, Lee, Das Sarma, PRA 81, 023602 (2010).

Magnetic dipoles

• Magnetic dipole moments are **quantum**, loosely speaking, proportional to hyperfine spin operators.

Permanent!



$$\vec{m}_{\alpha\beta} = g\mu_B \vec{F}_{\alpha\beta}$$

• Already spin eigenstates.

• No external magnetic fields are needed.

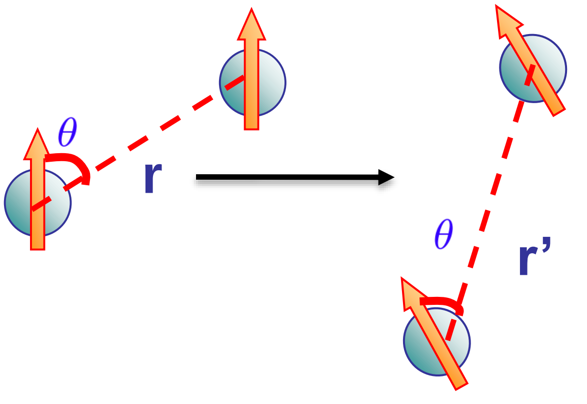
Fermion: ^{161}Dy, ^{163}Dy nearly quantum degenerate region [Lu et. al, 2010] (large magnetic moments ~ 10 μ_B)

• However, the energy scale of magnetic dipolar interactions are much weaker.

Spin-orbit (SO) coupled nature of the magnetic dipolar interaction

- Un-polarized magnetic dipolar fermions – our interest!
- Isotropy: invariant under the simultaneous **SO rotation.**

$$V(\vec{r}) = \frac{(g\mu)^2}{r^3}[\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \hat{r})(\vec{F}_2 \cdot \hat{r})]$$



- SO coupling at the interaction level, but not the single particle level!

1. SO coupled p-wave triplet unconventional cooper pairing, not another He-3.

2. SO coupled Fermi liquid collective excitations.

Magnetic dipolar gas:
Fermion: ^{161}Dy, ^{163}Dy nearly quantum degenerate region [Lu et. al, 2010] (large magnetic moments ~ 10

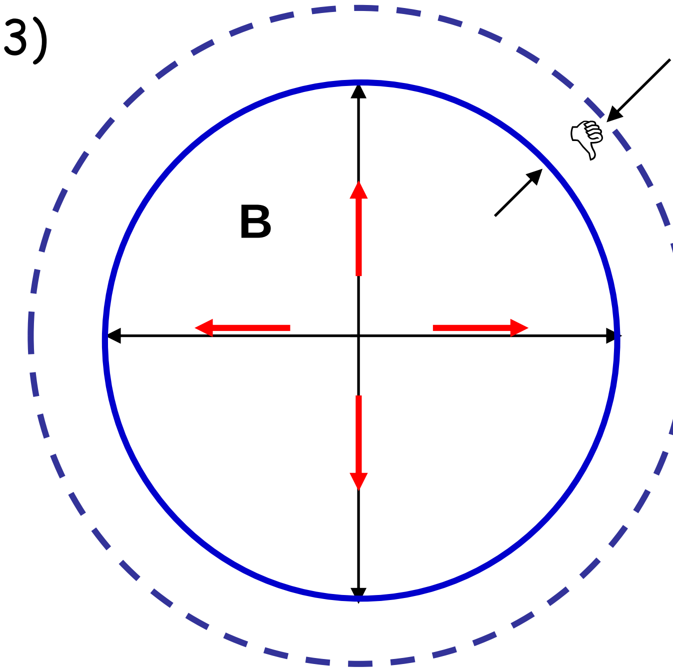
Review: p-wave (L=1) triplet (S=1) Cooper pairing of He-3

- Isotropic B phase. Time-reversal (TR) symm. preserved; J-singlet.

Balian-Werthamer (1963)

L=1, S=1, J=L+S=0.

$$\left(\frac{1}{\sqrt{2}}(p_x+ip_y)\,|\downarrow\downarrow\rangle + \frac{1}{\sqrt{2}}(p_x-ip_y)\,|\uparrow\uparrow\rangle + p_z\frac{1}{\sqrt{2}}\,|\uparrow\downarrow+\downarrow\uparrow\rangle\right)$$



- d-matrix for triplet Cooper pairing.

$$\Delta^*_{\alpha\beta}(\vec{k}) = \left\langle c^+_\alpha(\vec{k})c^+_\beta(-\vec{k})\right\rangle = |\Delta|\,(i\sigma_y\sigma_\mu)_{\alpha\beta}\,d_\mu(\vec{k})$$

$$= |\Delta| \begin{pmatrix} d_x + id_y & d_z \\ d_z & -d_x + id_y \end{pmatrix}$$

$$\hat{d}(\vec{k}) = \hat{k}$$

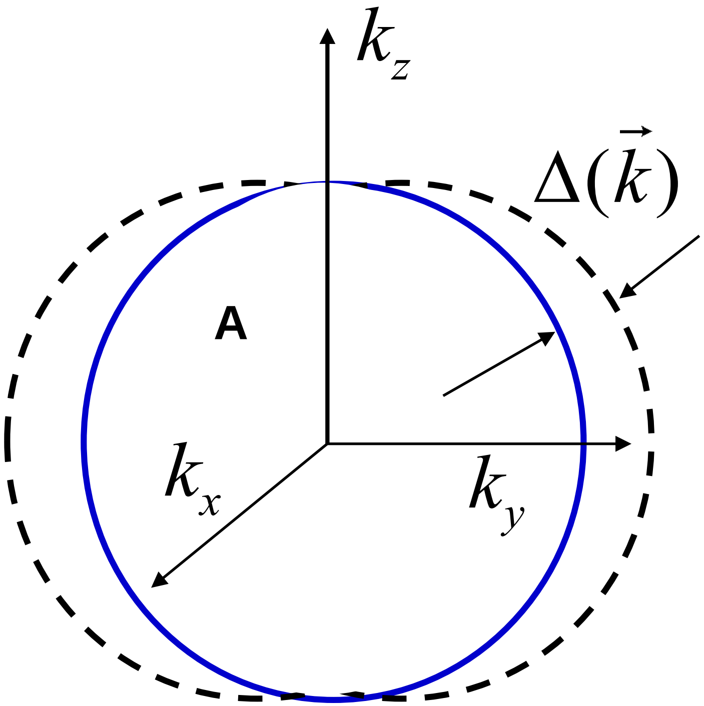
- Cooper pair spin fluctuates in the plane perpendicular to the d-vector.

Review: p-wave (L=1) triplet (S=1) Cooper pairing of He-3

- Anisotropic A phase. TR symm. breaking; J not well-defined.

Anderson-Brinkman-Morel (1961)

$(p_x + ip_y)\,(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$



$$\Delta^*_{\alpha\beta}(\vec{k}) = \left\langle c^+_\alpha(\vec{k})c^+_\beta(-\vec{k})\right\rangle = |\Delta|\,(i\sigma_y\sigma_\mu)_{\alpha\beta}\,d_\mu(\vec{k})$$

$$\vec{d}(\vec{k}) = \hat{z}(k_x + ik_y)$$

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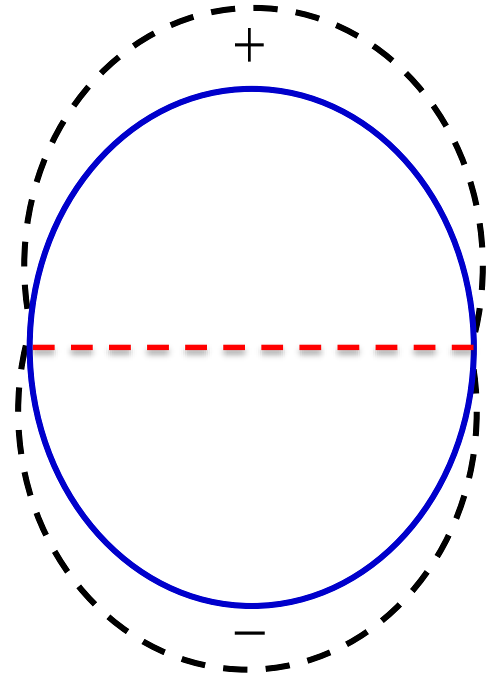
Single-componet electric dipolar fermions (pz pairing)

- Fermi surface uni-axial distotion.

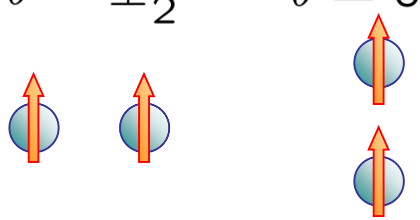
- p_z-wave-like pairing due to the anisotropy.



Baranov et al, PRA 66, 013606 (2002); PRL 92, 250403 (2004).

You et al 1999; G. M. Brunn 2008; Pu et al 2009.

$\theta = \pm\frac{\pi}{2}$ $\qquad$ $\theta = 0, \pi$



Repulsive **Attractive**

$$\Delta(\vec{k}) = \left\langle c^{+}(\vec{k})c^{+}(-\vec{k})\right\rangle \approx |\Delta| k_z$$

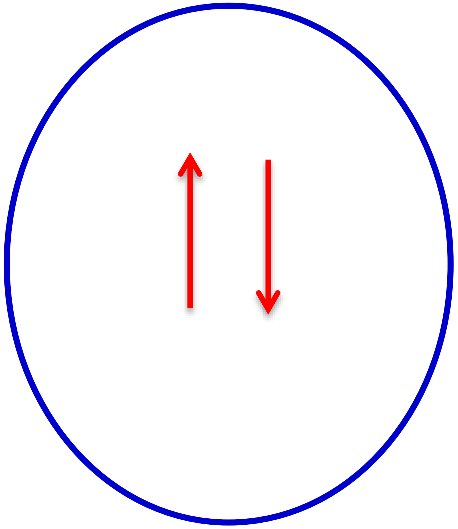
- Nodal plane: $k_z = 0$

Triplet pairing from electric dipolar pairing

• **Multi-component** Fermi gases with electric dipolar interactions. Both triplet and singlet pairings are allowed.

Competition: triplet p_z-channel (dominant) v.s. singlet s+d-wave channel (sub-dominant).

T. Shi et al, arxiv 0910.4051. C. Wu and J. E. Hirsch, PRB 81, 20508 (2010).



Mixing between triplet and singlet pairings leads to novel pairing breaking **time reversal symmetry!**

C. Wu and J. E. Hirsch, PRB 81, 20508 (2010).

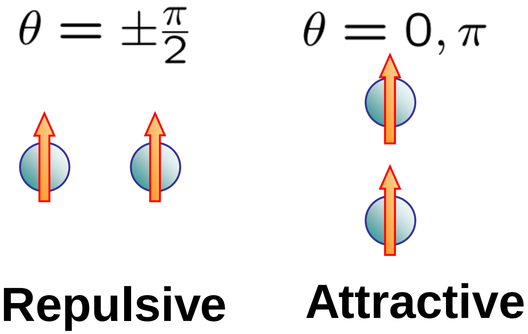
$$p_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$
$$+ i(s + d_{r^2-3z^2})(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Fourier transformation of electric dipolar interactions

- Real space:

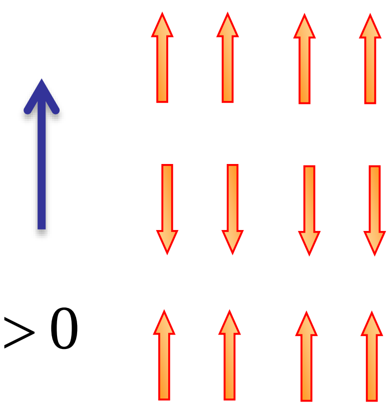
$$V(\vec{r}) = -\frac{d^2}{r^3} P_2(\cos\theta)$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$



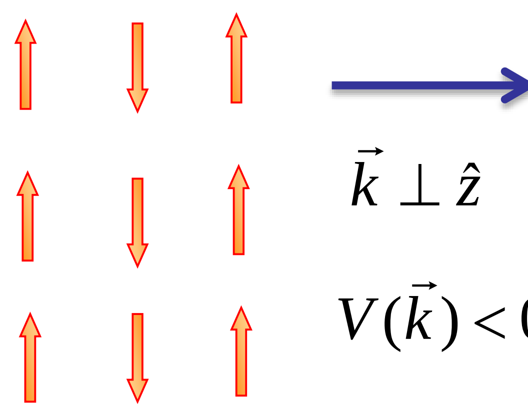
- Fourier transform: only depend on the direction of k with the same form of anisotropy.

$\vec{k} \,//\, \hat{z}$



$V(\vec{k}) > 0$

$$V_{diplr}(\vec{k}) = \frac{8\pi d^2}{3} P_2(\cos\theta_k)$$

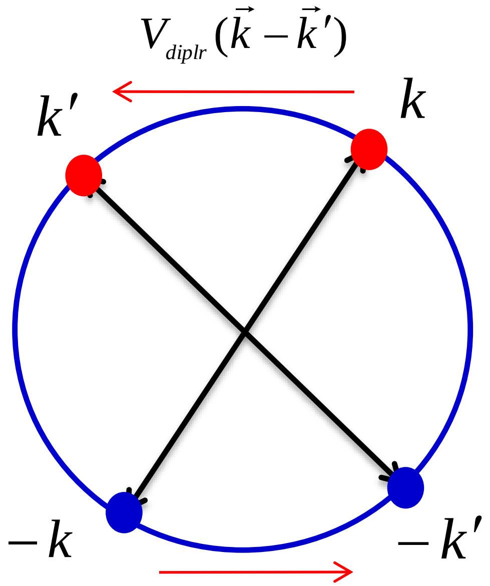


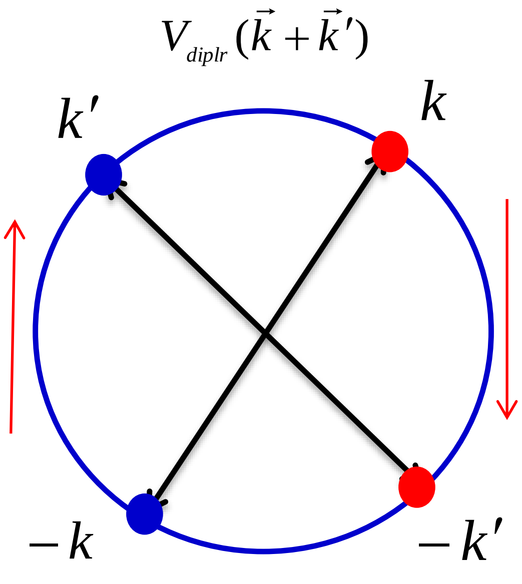
$\vec{k} \perp \hat{z}$

$V(\vec{k}) < 0$

Two-component **electric dipolar** fermions

$$H_{pair} = \frac{1}{2V}\sum_{k,k'}\{V_{tri}(k;k')\sum_{\mu=x,y,z}P_{tr}^{+,\mu}(k)P_{tr}^{\mu}(k') + V_{sg}(k;k')P_{sg}^{+}(k)P_{sg}(k')\}$$





$$V_{tri,sg}(\vec{k};\vec{k}') = \frac{1}{2}\{V_{dplr}(\vec{k}-\vec{k}') \mp V_{dplr}(\vec{k}+\vec{k}')\}$$

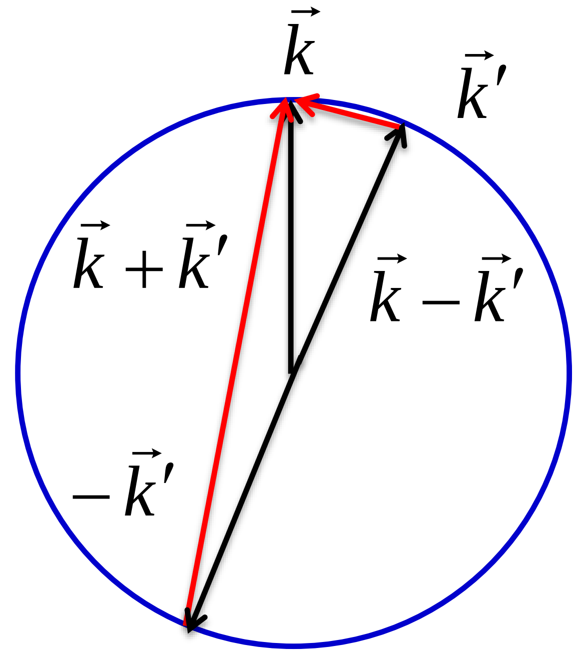
$$P_{sg}(k) = \tfrac{1}{\sqrt{2}}\{c^{+}_{k\uparrow}c^{+}_{-k\downarrow} - c^{+}_{k\downarrow}c^{+}_{-k\uparrow}\} \qquad P^{+}_{tr,z}(k) = \tfrac{1}{\sqrt{2}}\{c^{+}_{k\uparrow}c^{+}_{-k\downarrow} + c^{+}_{k\downarrow}c^{+}_{-k\uparrow}\}$$

$$P^{+}_{tr,x}(k) = \tfrac{1}{\sqrt{2}}\{-c^{+}_{k\uparrow}c^{+}_{-k\uparrow} + c^{+}_{k\downarrow}c^{+}_{-k\downarrow}\} \qquad P^{+}_{tr,y}(k) = \tfrac{i}{\sqrt{2}}\{c^{+}_{k\uparrow}c^{+}_{-k\uparrow} + c^{+}_{k\downarrow}c^{+}_{-k\downarrow}\}$$

Why pz-wave?

$$H^{tr}_{pair} = \frac{1}{2V}\sum_{k,k'}\{V_{tr}(k;k')\sum_{\mu=x,y,z}P^{+,\mu}_{tr}(k)P^{\mu}_{tr}(k')\}$$

$$V_{tr}(\vec{k};\vec{k}') = \frac{1}{2}\{V_{dplr}(\vec{k}-\vec{k}') - V_{dplr}(\vec{k}+\vec{k}')\}$$



- Take k along z-axis, and set k′ close to k.

$$(\vec{k}-\vec{k}') \perp \hat{z} \qquad (\vec{k}+\vec{k}') // \hat{z}$$

$$V_{dplr}(\vec{k}-\vec{k}') = -\frac{4\pi d^2}{3}, \quad V_{dplr}(\vec{k}+\vec{k}') = \frac{8\pi d^2}{3} \Rightarrow V_{tr}(\vec{k};\vec{k}') < 0$$

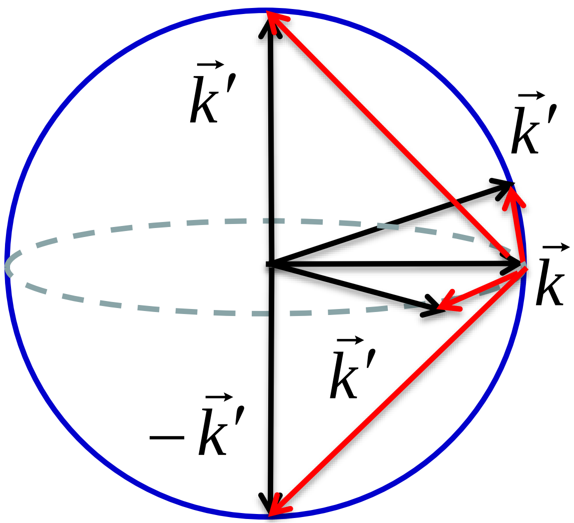
- Set k′ close to -k. $\Rightarrow$ $V_{tr}(\vec{k};\vec{k}') > 0$

- Dominate partial-wave component $V_{tr}(\vec{k};\vec{k}') \sim -\cos\theta_k \cos\theta_{k'}$

Singlet channel with purely dipolar interaction

$$H_{pair}^{sg} = \frac{1}{2V}\sum_{k,k'}\{V_{sg}(k;k')\sum_{\mu=x,y,z} P_{sg}^{+,\mu}(k)P_{tri}^{\mu}(k')\}$$

$$V_{sg}(\vec{k};\vec{k}') = \frac{1}{2}\{V_{dplr}(\vec{k}-\vec{k}')+V_{dplr}(\vec{k}+\vec{k}')\}$$



- k in the xy-plane, and set k' close to k.

$$V_{sg}(\vec{k};\vec{k}') = \begin{cases} \frac{2\pi d^2}{3} & \vec{k}'-\vec{k}:\text{longitude} \\ -\frac{4\pi d^2}{3} & \vec{k}'-\vec{k}:\text{latitude} \end{cases} \Rightarrow \bar{V}_{ag}(\vec{k};\vec{k}')<0$$

- k in the xy-plane, and set k' //z

$$\theta_{k+k'} = \theta_{k-k'} = \frac{\pi}{4} \quad V_{sg}(\vec{k};\vec{k}') = \frac{2\pi d^2}{3} > 0$$

- Sign changed → mostly d-wave, hybridized by s-wave due to anisotropy. Weaker than the pz-wave channel.

Two-component **electric dipolar** fermions

$$\Psi(\vec{k})=[c_\uparrow(\vec{k}),c_\downarrow(\vec{k}),c_\uparrow^\dagger(-\vec{k}),c_\downarrow^\dagger(-\vec{k})]^T.$$

$$H_{mf}=\sum_{\vec{k}}{}' \Psi^\dagger(\vec{k})\begin{pmatrix}\xi(\vec{k})I & \Delta_{\alpha\beta}(\vec{k})\\ \Delta^*_{\beta\alpha}(\vec{k}) & -\xi(\vec{k})I\end{pmatrix}\Psi(\vec{k}),$$

$$\Delta_{\alpha\beta}(k)=\Delta_{si}\phi^{s+d}(\Omega_k)i\sigma^y_{\alpha\beta}+\Delta^\mu_{tri}\phi^z(\Omega_k)i(\sigma^\mu\sigma^y)_{\alpha\beta}$$

- Gap function equations.

$$\Delta_{tri,\mu;si}(k)=-\int\frac{d^3k}{(2\pi)^3}V_{tri,si}(k;k')\{\frac{\tanh\frac{\beta E_{k'}}{2}}{2E_{k'}}-\frac{1}{2\varepsilon_{k'}}\}\Delta_{tri,\mu;si}(k')$$

Two-component **electric dipolar** fermions

- Linearized gap function equation, and partial wave analysis.

$$\frac{N_0}{4\pi} V_{tri,si}(k;k') = \sum_{ll';m} V_{ll';m} Y^*_{lm}(\Omega_k) Y_{l'm}(\Omega_{k'})$$

$l' = l, l \pm 2$ even for singlet pairing; odd for triplet pairing

$$T_c^j = \frac{2e^\gamma}{\pi} \overline{\omega} \exp(-1/|w^j_{tri,si}|)$$

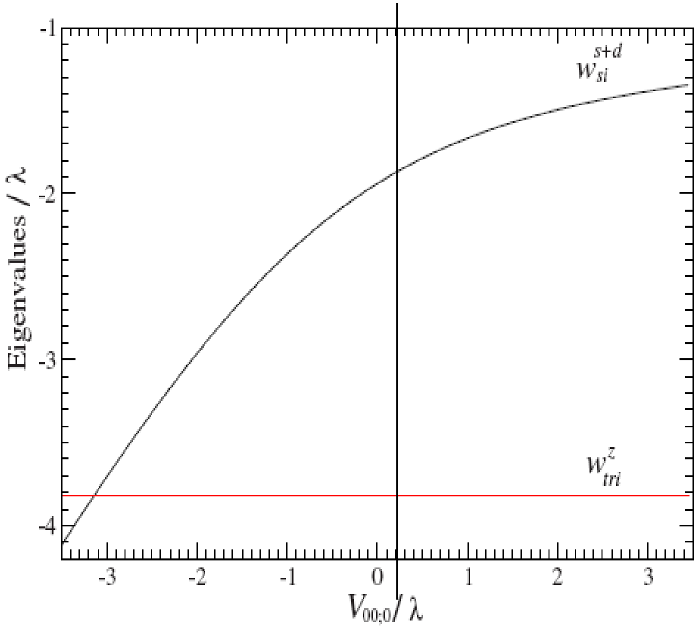
w^j : the most negative eigenvalue of $V_{ll';m}$

Baranov et al, PRA 66, 013606 (2002).

Dominant channels: triplet pz and singlet $s+d_{z^2}$

- V_0 : dimensionless short interaction strength

  $\lambda = E_{\text{int}} / E_K$ dimensionless dipolar interaction strength



- Triplet channel: mostly pz-wave. Insensitive to short-range interactions.

$$\phi^z(\Omega_k) \approx 0.99Y_{10} - 0.12Y_{30}; \quad w_{tri}^z = -3.82\lambda$$

- Singlet channel: hybridized s+d. Sensitive to short-range interactions.

$$\phi^{s+d}(\Omega_k) \approx 0.6Y_{00} - 0.8Y_{20}; \quad w_{si}^{s+d} = -1.95\lambda \text{ at } V_0 = 0$$

Dominant channels: triplet p_z and singlet $s+d_{r^2-3z^2}$

• Mixing between triplet and singlet pairing $V_0/\lambda=-3$. Weak coupling theory selects unitary paring → phase difference=$\pi/2$.

$$\Delta_{\alpha\beta}(k)=\Delta_{sg}\phi^{s+d}(\Omega_k)i\sigma^y_{\alpha\beta}+\Delta^\mu_{tri}\phi^z(\Omega_k)i(\sigma^\mu\sigma^y)_{\alpha\beta}$$

$$=\begin{bmatrix}(\Delta^x_{tr}+i\Delta^y_{tr})\phi^{s+d} & \Delta_{sg}\phi^{s+d}+\Delta^z_{tr}\phi^{s+d}\\ -\Delta_{sg}\phi^{s+d}+\Delta^z_{tr}\phi^{s+d} & (-\Delta^x_{tr}+i\Delta^y_{tr})\phi^{s+d}\end{bmatrix}\qquad \text{iff}\quad \theta_{tr}-\theta_{sg}=\pm\frac{\pi}{2},\quad \Delta^+(k)\Delta(k)\propto I$$

• A pairing state breaking time-reversal symmetry!

$$p_z(|k_\uparrow-k_\downarrow\rangle+|k_\downarrow-k_\uparrow\rangle)+i(s+d_{r^2-3z^2})(|k_\uparrow-k_\downarrow\rangle-|k_\downarrow-k_\uparrow\rangle)$$
$$=[p_z+i(s+d_{r^2-3z^2})]|k_\uparrow-k_\downarrow\rangle+[p_z-i(s+d_{r^2-3z^2})]|k_\downarrow-k_\uparrow\rangle$$

• Generalization to n-components – C. Wu and J. E. Hirsch, PRB 81, 20508 (2010).

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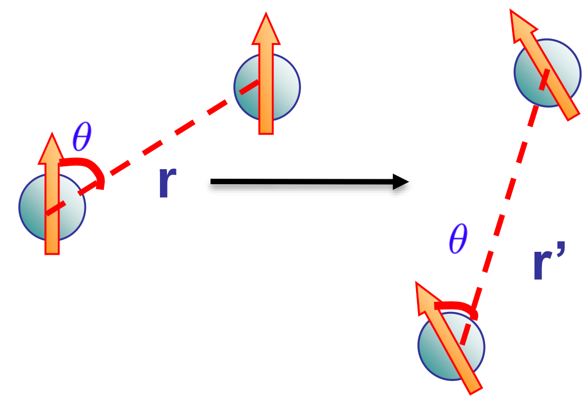

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2. SO coupled Fermi liquid collective excitations.

Magnetic dipolar gas:

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Spin-1/2 magnetic dipolar interaction

- The magnetic dipolar interaction vanishes in the total spin singlet channel $V_{\alpha\beta;\beta'\alpha'}\chi_{singlet} = 0$.

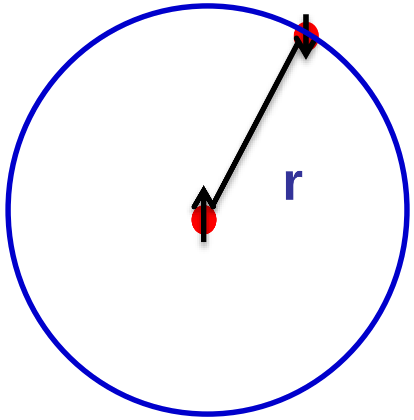
Here only exists spin triplet pairing in odd orbital partial wave channels.

$$[\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \hat{r})(\vec{F}_2 \cdot \hat{r})]\left|\uparrow_1\downarrow_2 - \downarrow_2\uparrow_1\right\rangle$$

$$= [-\frac{3}{4} - (-\frac{3}{4})]\left|\uparrow_1\downarrow_2 - \downarrow_2\uparrow_1\right\rangle = 0$$

Two-body Picture (I): p-wave spin triplet

- Consider a two-body problem with fixed inter particle radius d.



Consider L=1 channel and fix the distance between two particles:

	Total angular momentum		Interaction Energy
$S = 1$, $L = 1$	$J = 0$	→	+ E_{dp}
	$J = 1$	→	- 1/2 E_{dp}
	$J = 2$	→	+1/10 E_{dp}

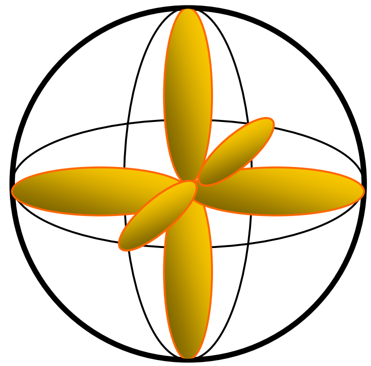
$$E_{dp} = \frac{\mu^2}{d^3}$$

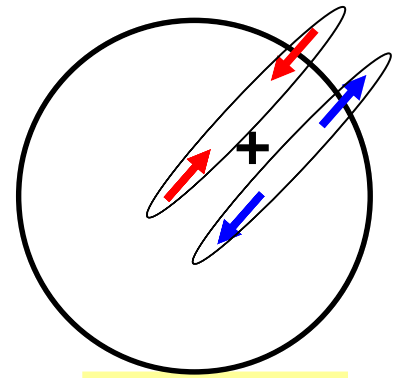
J-triplet channel gives bound states!

Two-body picture (II): spin Configurations

J-singlet state (J=0)

$$\phi_0 = \chi_x \widehat{p}_x + \chi_y \widehat{p}_y + \chi_z \widehat{p}_z$$





Repulsive

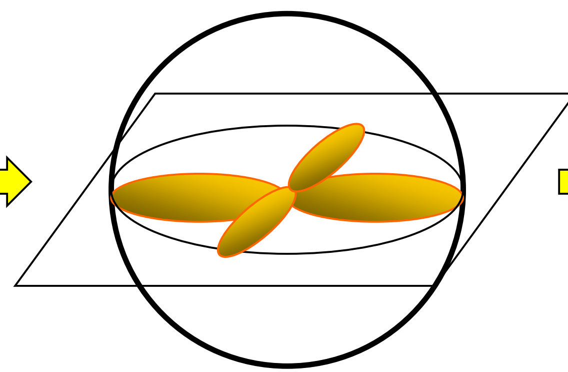
J-triplet state (J=1)

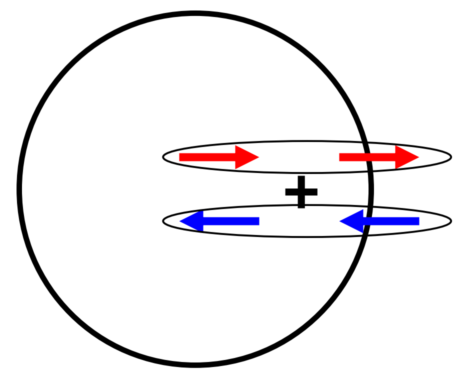
Eigenstate of Jz=0

$$\phi_z = \frac{1}{\sqrt{2}}(\chi_x \widehat{p}_y - \chi_y \widehat{p}_x)$$

$$\chi_x = \frac{1}{\sqrt{2}}(-|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\chi_y = \frac{i}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$





Attractive

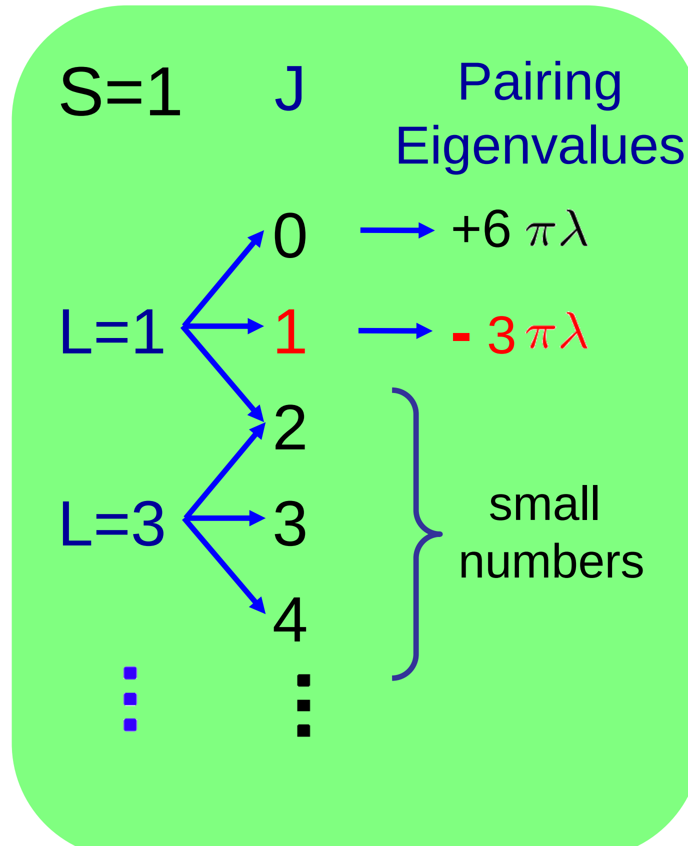
Momentum space partial-wave analysis (weak coupling)

$$H_{pair} = \frac{1}{2V}\sum_{k,k'}\{V_{\alpha\beta;\beta'\alpha'}(k;k')P_{\alpha\beta}^{+}(k)P_{\beta'\alpha'}(k')\}$$

$$\lambda = \frac{E_{int}}{E_F} = \frac{2}{3}\frac{\mu^2 m k_f}{\pi^2\hbar^2}$$

$$V_{\alpha\beta;\beta'\alpha'}(\vec{q}) = \frac{4\pi}{3}\mu^2[3(\vec{S}_{\alpha\alpha'}\cdot\hat{q})(\vec{S}_{\beta\beta'}\cdot\hat{q}) - \vec{S}_{\alpha\alpha'}\cdot\vec{S}_{\beta\beta'})]$$

- Pairing wave analysis shows the same J-triplet pairing: L=S=J=1.



- cf. He-3 B L=S=1, J=0

  He-3 A L=S=1, J is not conserved.

Polar v.s. axial pairing

- Competition among the J-triplet sector: polar (j_z=0) v.s axial (j_z=1) pairings $\vec{\Delta} = (\Delta_x, \Delta_y, \Delta_z)$.

$$F_{GL} = \alpha \vec{\Delta}^* \cdot \vec{\Delta} + \gamma_1 |\vec{\Delta}^* \cdot \vec{\Delta}|^2 + \gamma_2 |\vec{\Delta}^* \times \vec{\Delta}|^2$$

$\gamma_2 > 0 \Rightarrow \mathrm{Re}\vec{\Delta} // \mathrm{Im}\vec{\Delta}$ polar pairing state: TR symmetry maintained

$\gamma_2 < 0 \Rightarrow \mathrm{Re}\vec{\Delta} \perp \mathrm{Im}\vec{\Delta}$ axial pairing state: TR symmetry broken

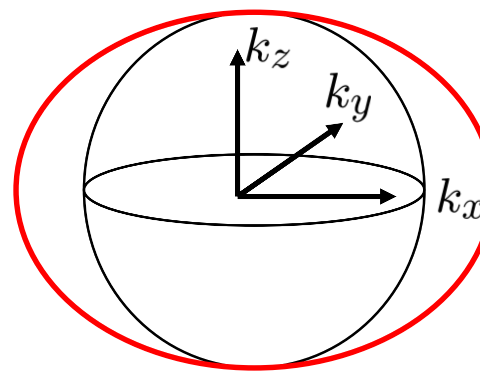
Helical polar pairing ($J=1;J_z=0$) v.s. Axial pairing ($J=J_z=1$)

- Helical polar pairing: TR invariant version of He-3 A phase.
- Unitary pairing.

$$\Delta_{\alpha\beta} \propto [(k_y\sigma_1 - k_x\sigma_2)i\sigma_2]_{\alpha\beta}$$

$$\propto \begin{bmatrix} -(\hat{k}_y + i\hat{k}_x) & 0 \\ 0 & \hat{k}_y - i\hat{k}_x \end{bmatrix}$$

$$\Delta^+\Delta \propto I$$



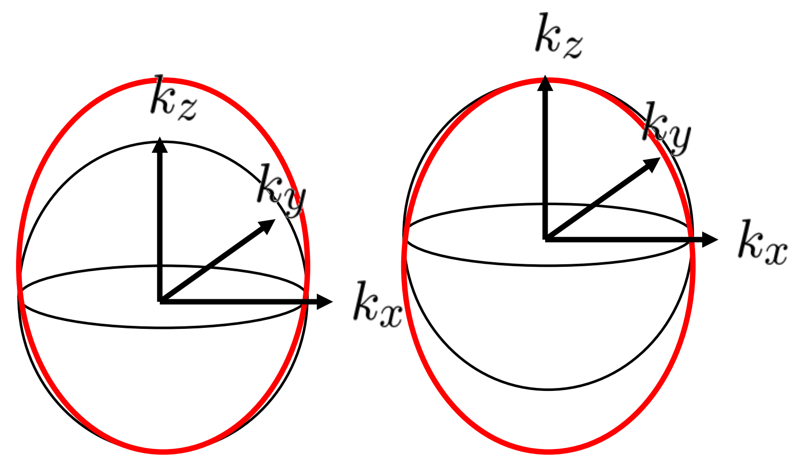
$$E_\alpha^{pl}(\mathbf{k}) = \sqrt{\xi_k^2 + \frac{1}{4}|\Delta|^2 \sin\theta_k^2}$$

Helical polar pairing (J=1;Jz=0) v.s. Axial pairing (J=Jz=1)

- Axial pairing: non-unitary pairing, TR symmetry breaking.

$$\Delta_{\alpha\beta} \propto [(k_z(\sigma_x + i\sigma_y) - \sigma_z(k_x + ik_y))i\sigma_2]_{\alpha\beta}$$

$$\propto \begin{bmatrix} \hat{k}_z & \frac{1}{2}(\hat{k}_x + i\hat{k}_y) \\ \frac{1}{2}(\hat{k}_x + i\hat{k}_y) & 0 \end{bmatrix}$$



$$\Delta\Delta^+ = |\Delta|^2 \left(\frac{k_z^2 + k^2}{2} + k_z(\vec{k}\cdot\vec{\sigma})\right)$$

$$\Delta^+\Delta = (\Delta\Delta^+)^*$$

$$H^2 = \begin{pmatrix} (\varepsilon_k - \mu)^2 + \Delta\Delta^+, & 0 \\ 0, & (\varepsilon_{-k} - \mu)^2 + \Delta^+\Delta \end{pmatrix}$$

$$\lambda^2_{1,2} \propto (k \pm k_z)^2 \propto (1 \pm \cos\theta_k)^2$$

$$E^{ax}_{1,2}(\mathbf{k}) = \sqrt{\xi_k^2 + \tfrac{1}{8}|\Delta|^2(1 \pm \cos\theta_k)^2}$$

Helical polar pairing (J=1;Jz=0) v.s. Axial pairing (J=Jz=1)

- Reduced 2*2 Hamiltonians for the gapless branch around the south pole.

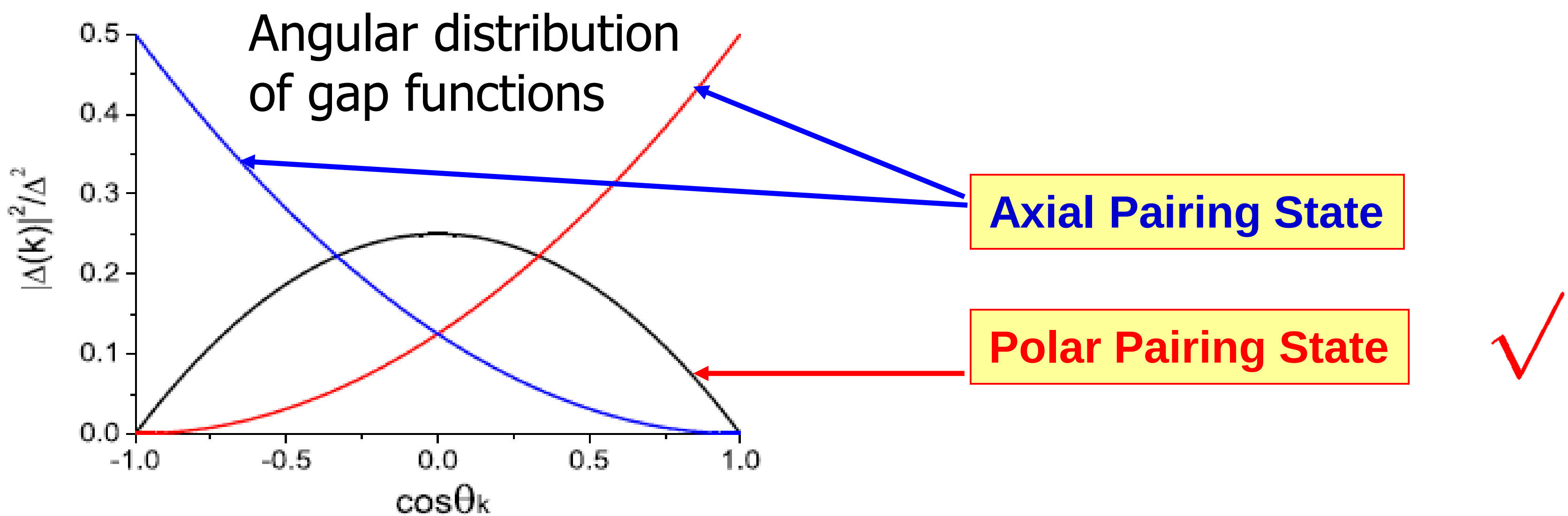
$$\psi_A = \frac{1}{\sqrt{2k(k-k_z)}}\begin{pmatrix} k_x - ik_y \\ k - k_z \\ 0 \\ 0 \end{pmatrix} \qquad \psi_B = \frac{1}{\sqrt{2k(k-k_z)}}\begin{pmatrix} 0 \\ 0 \\ k_x + ik_y \\ k - k_z \end{pmatrix}$$

$$H = \begin{bmatrix} v_f(k-k_f) & \frac{\sqrt{2}\Delta}{2k_f}(k_x+ik_y)^2 \\ \frac{\sqrt{2}\Delta}{2k_f}(k_x-ik_y)^2 & -v_f(k-k_f) \end{bmatrix}$$

- Weyl type fermion with winder number 2.

Which one is better? Polar pairing wins (BCS)!

• Polar pairing gap functions are more uniformly distributed over the Fermi surface. c.f. the competition of He3-B v.s. He3-A.



• We have numerically confirmed that polar pairing state has lower free energy at the BCS level.

• Strong coupling physics could stabilize the axial pairing. This possibility should not be ruled out!

Outline

• Difference between electric and magnetic dipolar interactions.

• Multi-component electric dipolar fermions: competition between triplet and singlet pairings.

Mixture between p-wave triplet and s+d single parings → Time-reversal symmetry breaking

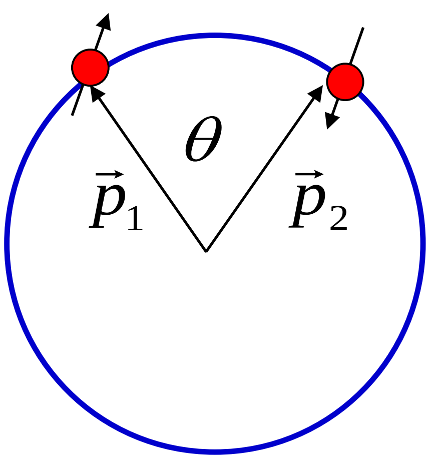
• Magnetic dipolar fermion systems: p-wave spin triplet with total angular momentum 1 (L=S=J=1)

A new pairing symmetry, not another He3.

• Fermi liquid: a "topological" collective mode of Fermi surface oscillation.

Landau Fermi liquid (FL) theory -- no SO coupling

L. Landau



- The existence of Fermi surface. Electrons close to Fermi surfaces are important → renormalized into quasi-particles.

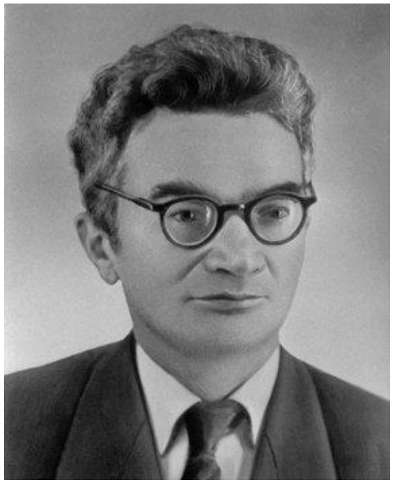
- Interaction functions (no SO coupling):

$$f_{\alpha\beta,\gamma\delta}(\hat{p}_1,\hat{p}_2) = f^s(\hat{p}_1,\hat{p}_2) \quad \text{density}$$
$$+ f^a(\hat{p}_1,\hat{p}_2)\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\gamma\delta} \quad \text{spin}$$

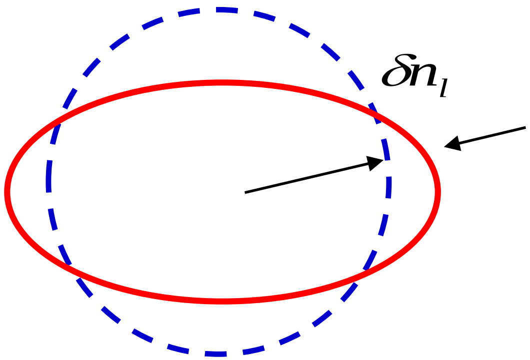
- Landau parameter in the *l*-th partial wave channel:

$$F_l^{s,a} = N_0 f_l^{s,a} \qquad N_0 : \text{DOS}$$

Pomeranchuk instability criterion



I. Pomeranchuk



- Fermi surface: elastic membrane.
- Stability:

$$\Delta E_K \propto (\delta n_l^{s,a})^2$$

$$\Delta E_{\text{int}} \propto \frac{F_l^{s,a}}{2l+1}(\delta n_l^{s,a})^2$$

- Surface tension vanishes at:

$$F_l^{s,a} < -(2l+1)$$

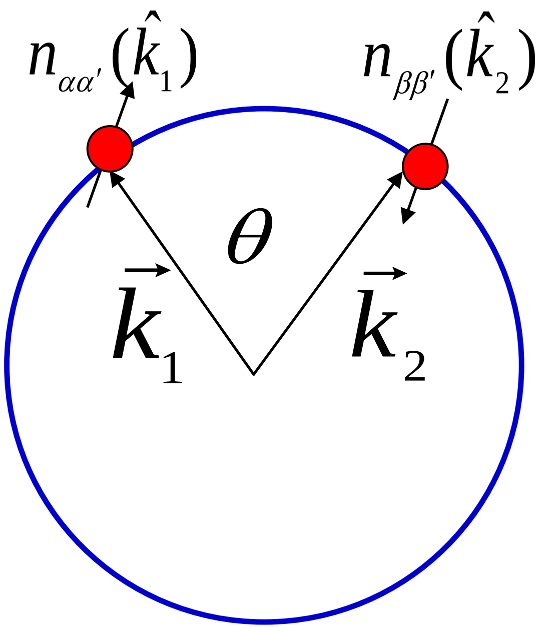
- Ferromagnetism: the F_0^a channel.
- Nematic electron liquid: the F_2^s channel.

- Unconventional magnetism: $F_l^a \ (l \geq 1)$

Spin-orbit coupled Fermi liquid theory

• Landau interaction functions: spin-orbit coupled partial-wave decomposition.

$$\frac{N_0}{4\pi} f_{\alpha\alpha',\beta\beta'}(\hat{k}_1,\hat{k}_2) = \sum_{JJ_zLL'} Y_{JJ_z;LS}(\hat{k}_1,\alpha\alpha') F_{JJ_zLS;JJ_zL'S} Y^{+}{}_{JJ_z;L'S}(\hat{k}_2,\beta\beta')$$



• Landau interaction matrices: $F_{JJ_zLS;JJ_zL'S}$

• Pomeranchuk instabilities.

• Criterion: there exists an eigenvalue of F <-1.

Pomeranchuk instability from magnetic dipolar interactions

- Leading channel: $J=1, J_z=\pm 1$, (even parity). Ferromagnetism with small hybridization with the ferronematic channel.

Fregoso et al PRL 2010.

- Sub-leading channel: $J=1, \quad J_z=0$, (odd parity). Transformation of SO coupling from the interaction level to the single particle level (Rashba like).

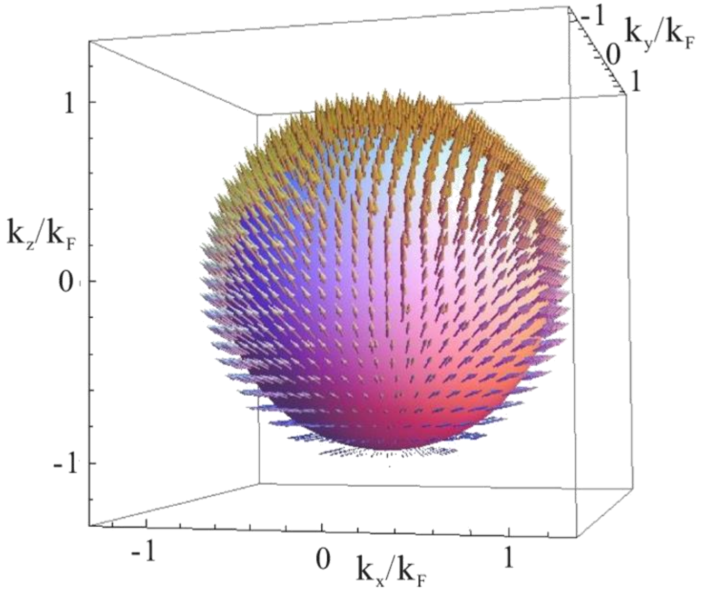
$$H_{so}(k) \propto k_x \sigma_y - k_y \sigma_x$$

Yi Li and Congjun Wu, Phys. Rev. B 85, 205126 (2012). Sogo et al, PRA 2012.

Topological zero-sound like collective modes

- Spin-orbit coupled Fermi surface oscillations.
  u_2: hedgehog distri; u_1: longitudinal ferro. u2>u1.

$$\vec{s}(\vec{r},\vec{k},t)=\begin{pmatrix} u_2 \sin\theta_{\vec{k}} \cos\phi_{\vec{k}} \\ u_2 \sin\theta_{\vec{k}} \sin\phi_{\vec{k}} \\ u_2 \cos\phi_{\vec{k}} + u_1 \end{pmatrix} e^{ik(z - s v_f t)}$$



- Sound velocity > Fermi velocity, under-damped

$$s_{\lambda\ll 1} \approx 1 + 2e^{-2(1+1/2F_+)} = 1 + 2e^{-2-12/7\pi\lambda}, \qquad s_{\lambda\gg 1} \approx \frac{F_\times}{3} = \frac{\pi}{3\sqrt{3}}\lambda.$$

Summary

• Magnetic dipole moment is quantum, while the electric one is classic.

• Magnetic dipolar interaction manifests spin-orbit coupled many-body physics.

• A novel spin-orbit coupled p-wave Cooper pairing is discovered, giving rise to total angular momentum J=1, which is NOT He-3 A and B.

Yi Li and Congjun Wu, Scientific Report 2, 392 (2012).

• Spin-orbit coupled Fermi liquid theory. An exotic spin-orbit coupled collective mode shows non-trivial spin configuration.

Yi Li and Congjun Wu, Phys. Rev. B 85, 205126 (2012).