In this supplementary material, we investigate thermodynamic quantities including compressibility and nearest-neighbor spin-spin correlations. These quantities, though not directly related with the Pomeranchuk cooling, are of direct interest in current experiments in ultracold atom physics. They provide a comprehensive understanding of thermodynamical properties of the SU(2N) Hubbard model at half-filling.

**COMPRESSIBILITY**

The compressibility $\kappa$ can be expressed in terms of the global charge fluctuations as

$$\kappa_{su(2N)} = \frac{1}{T L^2} \frac{\partial N_f}{\partial \mu} = \frac{1}{T L^2} (\langle \hat{N}_f^2 \rangle - \langle \hat{N}_f \rangle^2),$$

(1)

where $\hat{N}_f = \sum_i \hat{n}_i$ is the total fermion number operator in the lattice; $\mu$ is the chemical potential. In Fig. 1, we plot the simulated results for the normalized $\kappa_{su(2N)}/N$, i.e., the contribution to $\kappa_{su(2N)}$ per fermion component. They behave similarly to each other. $\kappa_{su(2N)}$ scales as $1/T$ like ideal gas at high temperatures, while they are suppressed at low temperatures. At zero temperature, $\kappa_{su(2N)}$ is suppressed to zero due to the charge gap in the Mott-insulating states. $\kappa_{su(2N)}$ reaches the maximum at an intermediate temperature scale which can be attributed to the energy scale of charge fluctuations.

**SPIN SUSCEPTIBILITY**

At finite temperatures, no magnetic long-range-order should exist in the 2D half-filled SU(2N) model due to its continuous symmetry. The normalized uniform SU(2N) spin susceptibility is defined as

$$\chi_{su(2N)}(T) = \frac{\beta}{N L^2} \sum_{i,j} M_{spin}(i,j).$$

(2)

The DQMC simulation results are presented in Fig. 2 for $U/t = 4$. At high temperatures, $\chi_{su(2N)}$ exhibits the standard Curie-Weiss law which scales proportional to $1/T$. $\chi_{su(2N)}$ reaches the maximum at an intermediate temperature at the scale of $J$ below which $\chi_{su(2N)}$ is suppressed by the AF exchange. At the lowest temperature we simulated, we did not observe the suppressions of $\chi_{su(2N)}$ for $2N = 4$ and 6. The nature of the ground states of half-filled SU(2N) Hubbard model remains an
open question in literatures when $2N$ is small but larger than 2. Nevertheless, we expect that they are either AF long-range-ordered like the case of $SU(2)$, or quantum paramagnetic with or without spin gap like in the large-$N$ limit. In either case, $\chi_{su(2N)}$ should be suppressed to zero with approaching zero temperature.

THE NEAREST-NEIGHBOR SPIN-SPIN CORRELATION

The nearest-neighbor (NN) spin-spin correlation in the $SU(2N)$ Hubbard model is defined as:

$$M_{spin, NN} = \frac{1}{(2N)^2 - 1} \sum_{\alpha, \beta} (S_{\alpha \beta, i} S_{\beta \alpha, j}),$$

where $i$ and $j$ are two nearest-neighbor lattice sites, and $S_{\alpha \beta, i} = c_{\alpha, i}^\dagger c_{\beta, i} - \frac{1}{2N} \delta_{\alpha \beta} n_i$. For the $SU(2)$ Hubbard model, the NN correlations have been probed recently using the lattice modulation technique [1]. The NN spin-spin correlations v.s. $T/t$ for fixed $U/t$ and different $2N$ have been plotted in Fig. 3. Notice that the monotonic behavior of NN spin-spin correlations as a function of $T$ indicates that these quantities can be used to measure temperatures and entropy in the Mott-insulating states.

SPIN-SPIN CORRELATIONS IN REAL SPACE

In Fig. 4, we plot the renormalized equal time spin-spin correlations for the $SU(2N)$ Hubbard model as a function of distance defined as

$$M_{spin}(r) = \frac{1}{(2N)^2 - 1} \sum_{\alpha, \beta} (S_{\alpha \beta}(1) S_{\beta \alpha}(1 + re_x)),$$

which exhibit a staggered antiferromagnetic structure. For the case of $SU(4)$ and $SU(6)$, spin-spin correlation functions decay much more drastically than that of $SU(2)$. This agrees with the fact that the AF correlations of the $SU(2N)$ Hubbard model are weaken with increasing $2N$.

THE CHARGE GAP

The charge gap is defined as the energy cost to add one particle in the ground state of the system composed of $N$-particles. Assume that

$$\hat{H} |\Psi_0^{N+1}\rangle = E_0^{N+1} |\Psi_0^{N+1}\rangle,$$

$$\hat{H} |\Psi_0^{N}\rangle = E_0^N |\Psi_0^{N}\rangle,$$

where $\hat{H}$ is the Hamiltonian of the grand canonical ensemble for the $SU(2N)$ Hubbard model as Eq. (1) in the main text. (The chemical potential $\mu$ is set 0 in Eq. (1)). The charge gap is $\Delta_c = E_0^{N+1} - E_0^N$. The onsite time-displaced Green’s function for $\tau > 0$ reads

$$G^>(\vec{r} = 0, \tau) = \frac{1}{L^2} \sum_i G^>(\tau)_{ii} = \frac{1}{L^2} \sum_i \langle \Psi_0^{N} | e^{\tau \hat{H}} c_i e^{-\tau \hat{H}} c_i^\dagger | \Psi_0^{N}\rangle.$$

FIG. 3: The normalized NN spin-spin correlation v.s. $T/t$ at $U/t = 4$ for $2N = 2, 4,$ and 6.

FIG. 4: The normalized spin-spin correlations as functions of distance (along the x-axis) with $T/t = 0.1$, $U/t = 4$ and $2N = 2, 4,$ and 6.
By inserting the complete set $I = \sum_n |\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|$, the above equation becomes

$$G^>(0, \tau) = \frac{1}{L^2} \sum_{i,n} e^{-\tau(E_{n}^{N+1} - E_{0}^{N})} \langle\Psi_0^N|c_i|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|c_i^\dagger|\Psi_0^N\rangle = \frac{1}{L^2} \sum_{i,n} e^{-\tau(E_{n}^{N+1} - E_{0}^{N})} |\langle\Psi_0^N|c_i|\Psi_n^{N+1}\rangle|^2. \quad (6)$$

Therefore, at large $\tau$, we have $G^>(\vec{r} = 0, \tau) \sim e^{-\tau\Delta_c}$ which can be used to estimate the value of $\Delta_c$ [2].
