Supplementary material for Pomeranchuk cooling of the SU($2N$) ultra-cold fermions in optical lattices

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In this supplementary material, we investigate thermodynamic quantities including compressibility and nearest-neighbor spin-spin correlations. These quantities, though not directly related with the Pomeranchuk cooling, are of direct interests in current experiments in ultracold atom physics. They provide a comprehensive understanding of thermodynamical properties of the SU($2N$) Hubbard model at half-filling.

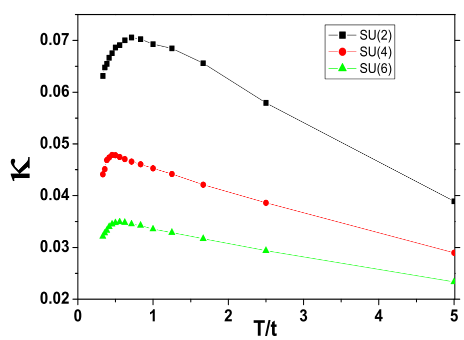

FIG. 1: The normalized compressibility $\kappa_{su(2N)}/(2N)$ v.s. T at $U/t = 4$ for $2N = 2, 4$, and 6.

COMPRESSIBILITY

The compressibility κ can be expressed in terms of the global charge fluctuations as

$$\kappa_{su(2N)} = \frac{1}{L^2}\frac{\partial N_f}{\partial \mu} = \frac{1}{TL^2}(\langle \hat{N}_f^2\rangle - \langle \hat{N}_f\rangle^2), \qquad (1)$$

where $\hat{N}_f = \sum_i \hat{n}_i$ is the total fermion number operator in the lattice; μ is the chemical potential. In Fig. 1, we plot the simulated results for the normalized $\kappa_{su(2N)}/N$, i.e., the contribution to $\kappa_{su(2N)}$ per fermion component. They behave similarly to each other. $\kappa_{su(2N)}$ scales as $1/T$ like ideal gas at high temperatures, while they are suppressed at low temperatures. At zero temperature, $\kappa_{su(2N)}$ is suppressed to zero due to the charge gap in the Mott-insulating states. $\kappa_{su(2N)}$ reaches the maximum at an intermediate temperature scale which can be attributed to the energy scale of charge fluctuations.

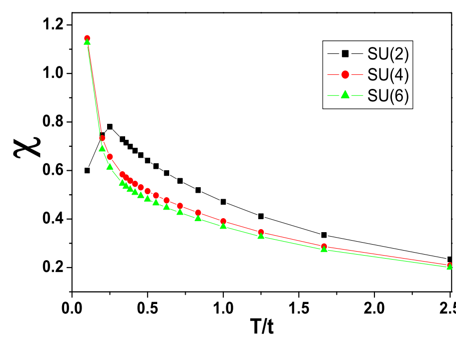

FIG. 2: The normalized SU($2N$) susceptibilities $\chi_{su(2N)}$ v.s. T with fixed $U/t = 4$ for $2N = 2, 4$, and 6.

SPIN SUSCEPTIBILITY

At finite temperatures, no magnetic long-range-order should exist in the 2D half-filled SU($2N$) model due to its continuous symmetry. The normalized uniform SU($2N$) spin susceptibility is defined as

$$\chi_{su(2N)}(T) = \frac{\beta}{NL^2}\sum_{\vec{i},\vec{j}} M_{spin}(i,j). \qquad (2)$$

The DQMC simulation results are presented in Fig. 2 for $U/t = 4$. At high temperatures, $\chi_{su(2N)}$ exhibits the standard Curie-Weiss law which scales proportional to $1/T$. $\chi_{su(2)}$ reaches the maximum at an intermediate temperature at the scale of J below which $\chi_{su(2)}$ is suppressed by the AF exchange. At the lowest temperature we simulated, we did not observe the suppressions of $\chi_{su(2N)}$ for $2N = 4$ and 6. The nature of the ground states of half-filled $SU(2N)$ Hubbard model remains an open question in literatures when $2N$ is small but larger than 2. Nevertheless, we expect that they are either AF long-range-ordered like the case of $SU(2)$, or quantum paramagnetic with or without spin gap like in the large-N limit. In either case, $\chi_{su(2N)}$ should be suppressed to zero with approaching zero temperature.

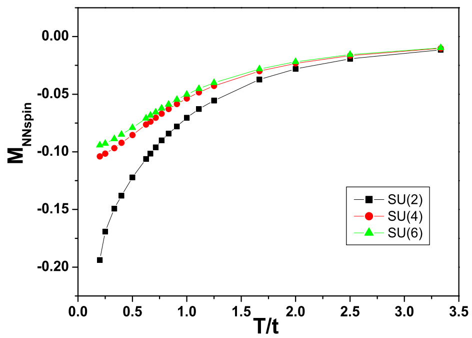

FIG. 3: The normalized NN spin-spin correlation v.s. T/t at $U/t = 4$ for $2N = 2, 4,$ and 6.

THE NEAREST-NEIGHBOR SPIN-SPIN CORRELATION

The nearest-neighbor (NN) spin-spin correlation in the SU($2N$) Hubbard model is defined as:

$$M_{spinNN} = \frac{1}{(2N)^2 - 1} \sum_{\alpha,\beta} \langle S_{\alpha\beta,i} S_{\beta\alpha,j} \rangle, \qquad (3)$$

where i and j are two nearest-neighbor lattice sites, and $S_{\alpha\beta,i} = c^\dagger_{\alpha,i} c_{\beta,i} - \frac{1}{2N}\delta_{\alpha\beta} n_i$. For the SU(2) Hubbard model, the NN correlations have been probed recently using the lattice modulation technique [1]. The NN spin-spin correlations v.s. T/t for fixed U/t and different $2N$ have been plotted in Fig. 3. Notice that the monotonic behavior of NN spin-spin correlations as a function of T indicates that these quantities can be used to measure temperatures and entropy in the Mott-insulating states.

SPIN-SPIN CORRELATIONS IN REAL SPACE

In Fig.4, we plot the renormalized equal time spin-spin correlations for the SU($2N$) Hubbard model as a function of distance defined as

$$M_{spin}(r) = \frac{1}{(2N)^2 - 1} \langle \sum_{\alpha,\beta} S_{\alpha\beta}(\mathbf{i}) S_{\beta\alpha}(\mathbf{i} + r\mathbf{e_x}) \rangle, \qquad (4)$$

which exhibit a staggered antiferromagnetic structure. For the case of SU(4) and SU(6), spin-spin correlation functions decay much more drastically than that of SU(2). This agrees with the fact that the AF correlations of the SU($2N$) Hubbard model are weaken with increasing $2N$.

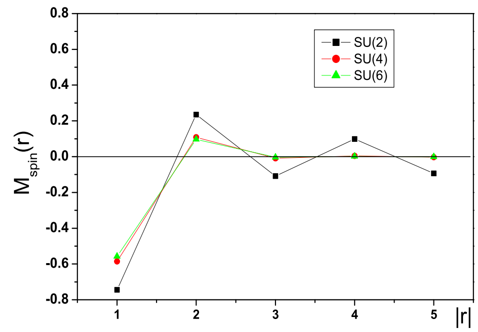

FIG. 4: The normalized spin-spin correlations as functions of distance (along the x-axis) with $T/t = 0.1$, $U/t = 4$ and $2N = 2, 4,$ and 6.

THE CHARGE GAP

The charge gap is defined as the energy cost to add one particle in the ground state of the system composed of N-particles. Assume that

$$\begin{aligned} \hat{H}|\Psi_0^{N+1}\rangle &= E_0^{N+1}|\Psi_n^{N+1}\rangle, \\ \hat{H}|\Psi_0^{N}\rangle &= E_0^{N}|\Psi_n^{N}\rangle, \end{aligned} \qquad (5)$$

where $\hat{H}$ is the Hamiltonian of the grand canonical ensemble for the SU($2N$) Hubbard model as Eq. (1) in the main text. (The chemical potential μ is set 0 in Eq. (1)). The charge gap is $\Delta_c = E_0^{N+1} - E_0^N$. The onsite time-displaced Green's function for $\tau > 0$ reads

$$G^>(\vec{r} = 0, \tau) = \frac{1}{L^2} \sum_i G^>(\tau)_{ii} = \frac{1}{L^2} \sum_i \langle \Psi_0^N | e^{\tau \hat{H}} c_i e^{-\tau \hat{H}} c_i^\dagger | \Psi_0^N \rangle.$$

By inserting the complete set $I = \sum_n |\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|$, the above equation becomes

$$G^{>}(0,\tau) = \frac{1}{L^2}\sum_{i,n} e^{-\tau(E_n^{N+1}-E_0^N)}\langle\Psi_0^N|c_i|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|c_i^\dagger|\Psi_0^N\rangle = \frac{1}{L^2}\sum_{i,n} e^{-\tau(E_n^{N+1}-E_0^N)}|\langle\Psi_0^N|c_i|\Psi_n^{N+1}\rangle|^2. \tag{6}$$

Therefore, at large τ, we have $G^{>}(\vec{r}=0,\tau) \sim e^{-\tau\Delta_c}$ which can be used to estimate the value of Δ_c [2].

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