Interaction effects in topological systems: helical Luttinger edge liquid and Majorana flat band

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Selected Reference

1. C. Wu, B. Andrei Bernevig, and S. C. Zhang, Phys. Rev. Lett. 96 , 106401(2006).
2. Dong Zheng, Guang-Ming Zhang, C. Wu, Phys. Rev. B 84, 205121 (2011).
3. Da Wang, Zhou-Shen Huang, Congjun Wu, Phys. Rev. B 89, 174510 (2014) .
4. Yi Li, Da Wang, Congjun Wu, New J. Phys 15, 085002(2013)

Inst. of Semi-cond, July 29, 2016

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Supported by NSF, AFOSR

Outline

- Introduction.

Quantum anomalous Hall → topological insulators (TI) → topological superconductor

- Stability criterial of interacting edge states of 2D TIs, and QMC study of interacting 2D TIs.

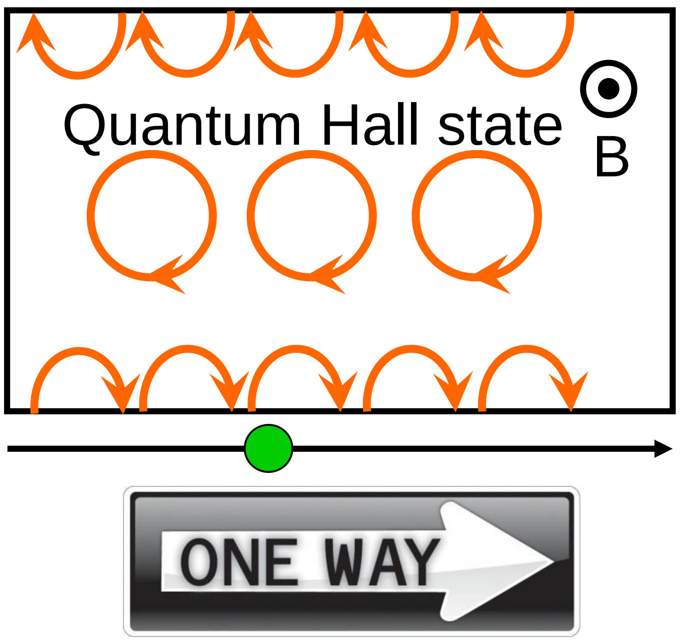
Helical Luttinger liquid and its stability against strong interactions.

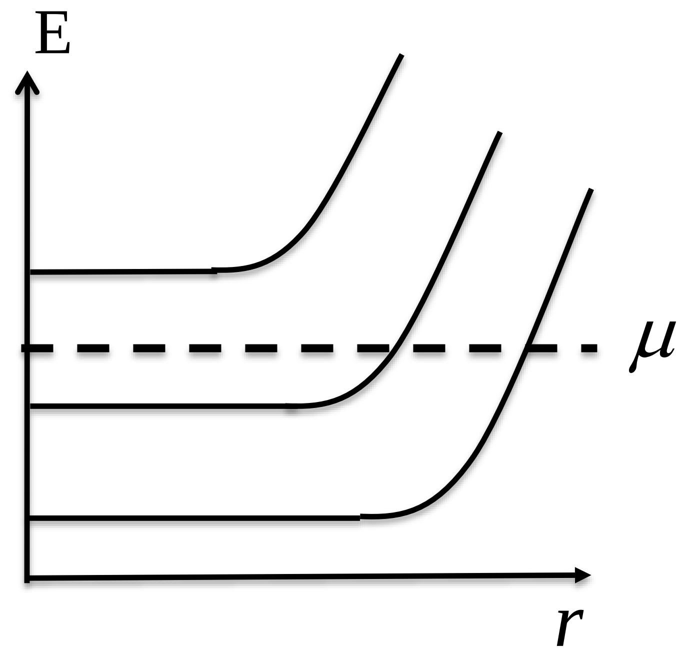
- Spontaneous time-reversal symmetry breaking in Majorana flat-bands.

2D quantum hall systems

• Chiral edge modes responsible for quantized transverse charge transport; stable against disorder and interactions.

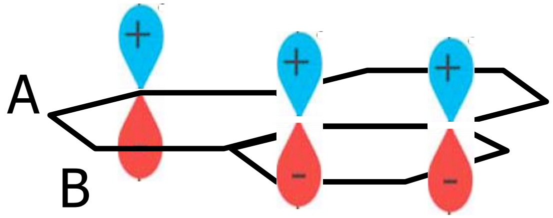
• Magnetic band-structure characterized by the topological TKNN (Chern) number.



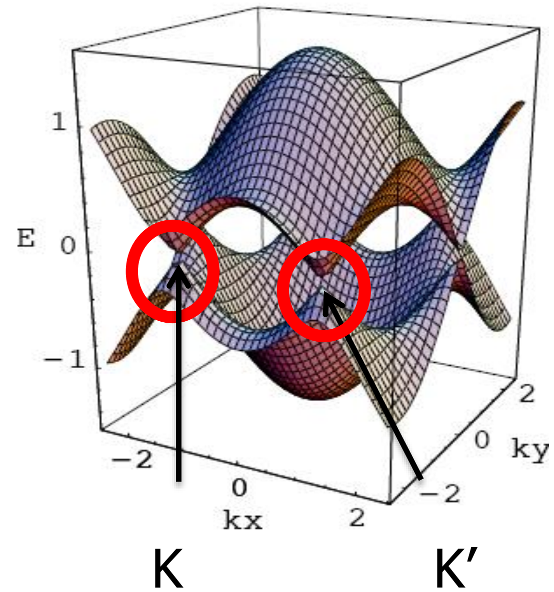


Halperin, PRB 25, 2185 (1982).

Honeycomb lattice system (graphene)



$$\psi(k) = \begin{pmatrix} c_A(k) \\ c_B(k) \end{pmatrix}$$

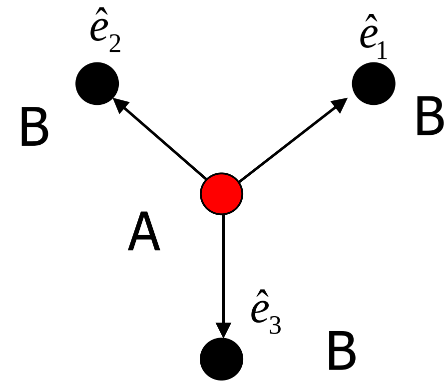


- 2-component spinor: A-B → two-level with a pseudo-B field.

$$H(\vec{k}) = \vec{h}(\vec{k}) \cdot \vec{\tau}$$

- h(k) is planar.

$$h_x(\vec{k}) = \sum_{i=1}^{3} \cos \vec{k} \cdot \hat{e}_i \quad h_y(\vec{k}) = \sum_{i=1}^{3} \sin \vec{k} \cdot \hat{e}_i$$

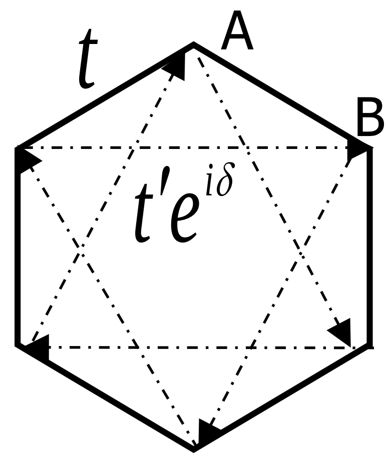


- Gapless Dirac cones ←→ vanishing of h(k); protected by symmetry and topology.

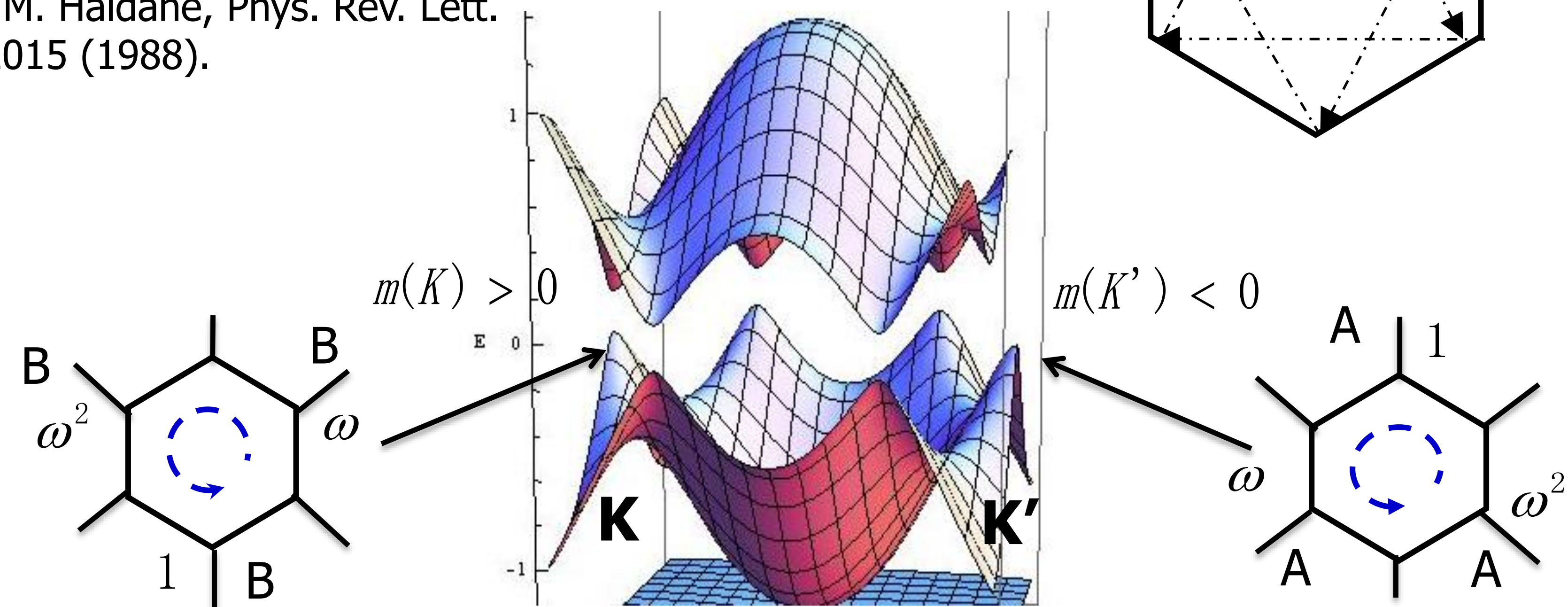
The Haldane model – complex NNN hopping

- TR breaking → Mass term flips the sign at K and K'. Work at UCSD!

$$h_z(\vec{k}) = m(\vec{k}) = t' \sin\delta \sum_{ij} \sin \vec{k} \cdot (\hat{e}_i - \hat{e}_j)$$



E. Fradkin, PRL 57, 2967 (1986)
F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).



Anomalous Hall effect (AHE) and its quantization

- Berry phase and curvature in momentum space.

$$\vec{A}(k) = \langle \psi_{L,k} \mid i\vec{\partial}_k \mid \psi_{L,k} \rangle, \qquad \Omega_z(\vec{k}) = \partial_{k_x} A_{k_y} - \partial_{k_y} A_{k_x}$$

- Anomalous velocity.

Luttinger, PR, 112 793 (1958), Xiao, Chang, Niu, RMP 82, 1959 (2010).

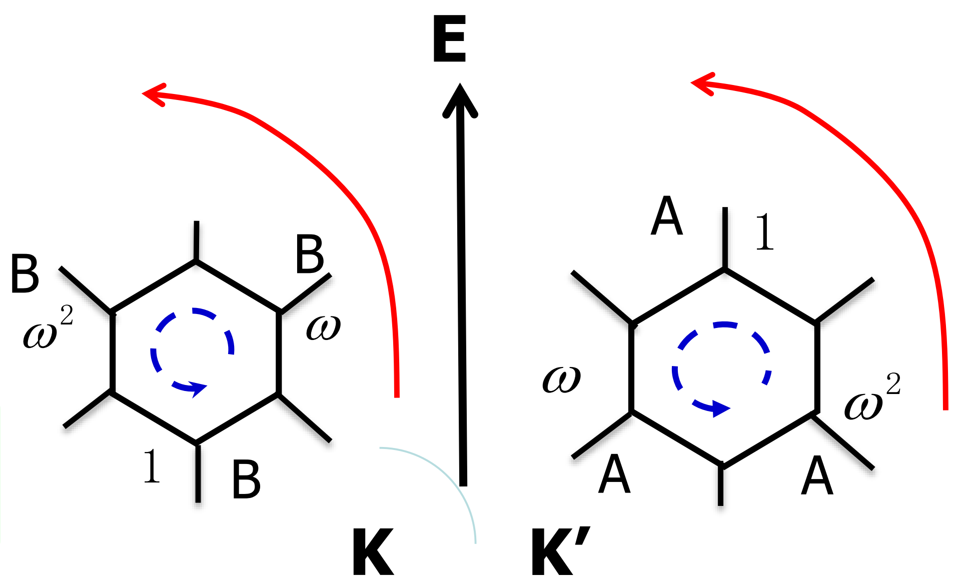
$$\dot{\vec{r}} = \nabla_k \varepsilon(\vec{k}) - \dot{\vec{k}} \times \vec{\Omega}(\vec{k})$$

$$\dot{\vec{k}} = q\vec{E} + q\dot{\vec{r}} \times \vec{B}(\vec{r})$$

$$\Omega_z(K) = \Omega_z(K')$$

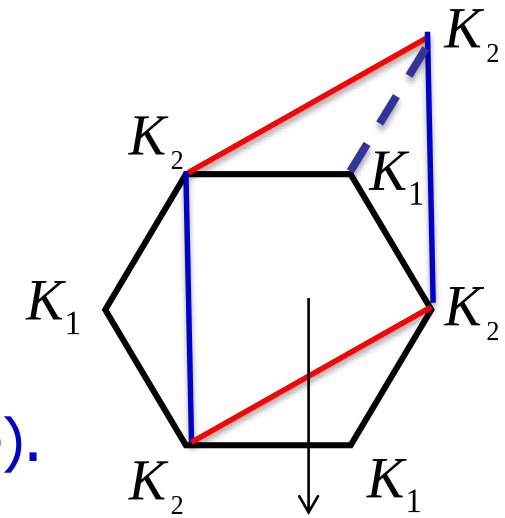
- Inter-band Van Vleck type response.

$$\sigma_{xy} = \frac{e^2}{h} \int \frac{dk_x dk_y}{2\pi} \Omega_z(\vec{k}) n_f(\vec{k})$$



Hall conductance as topological Chern (TKNN) number

- Brillouin zone has no boundary – torus.



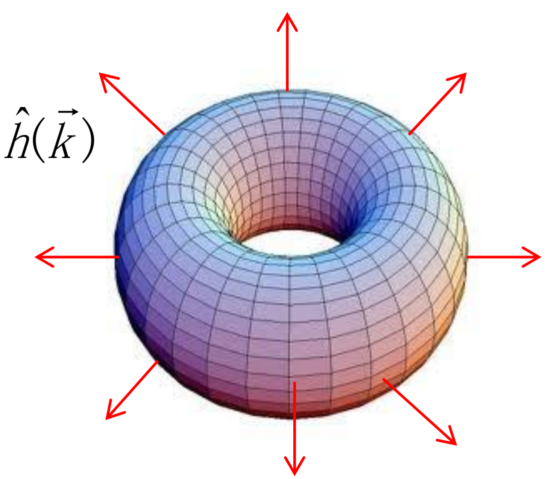
$$\nu = \oiint \frac{d^2\vec{k}}{2\pi}\,\Omega_z(\vec{k}) = \oiint \frac{d^2\vec{k}}{2\pi}\left(\nabla \times \vec{A}(\vec{k})\right) = \pm 1$$

- Topology in the Kubo formula (linear response).

$$\sigma_{xy} = \lim_{\omega\to 0}\lim_{q\to 0}\frac{i}{\omega}\,\mathrm{Im} < j_x(q,\omega)\, j_y(-q,-\omega) >$$

$$= \frac{e^2}{8\pi^2\hbar}\int\int dk_x dk_y\, \hat{h}\cdot(\partial_{k_x}\hat{h}\times\partial_{k_y}\hat{h}) =$$

$$= \frac{e^2}{2\pi\hbar}\int \frac{d^2\vec{k}}{2\pi}\Omega_z(\vec{k}) = \nu\,\frac{e^2}{h}$$

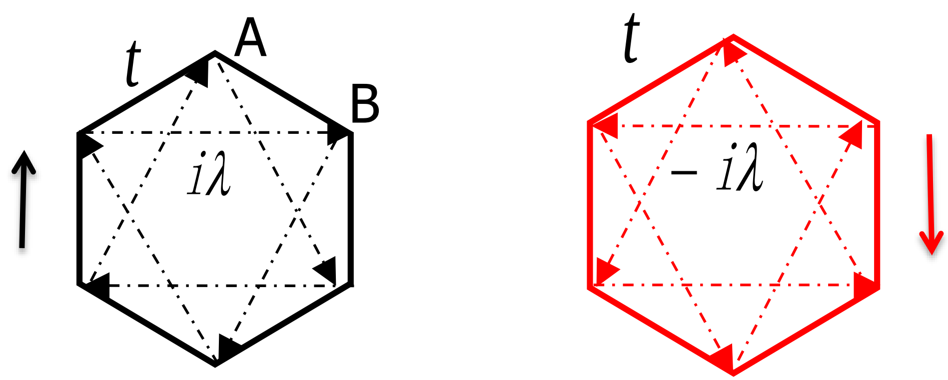


Thouless, Kohmoto, Nightingale, den Nijis, PRL 49, 405 (1982)

Exp: C. Z. Chang et al, Science 340, 167 (2013)

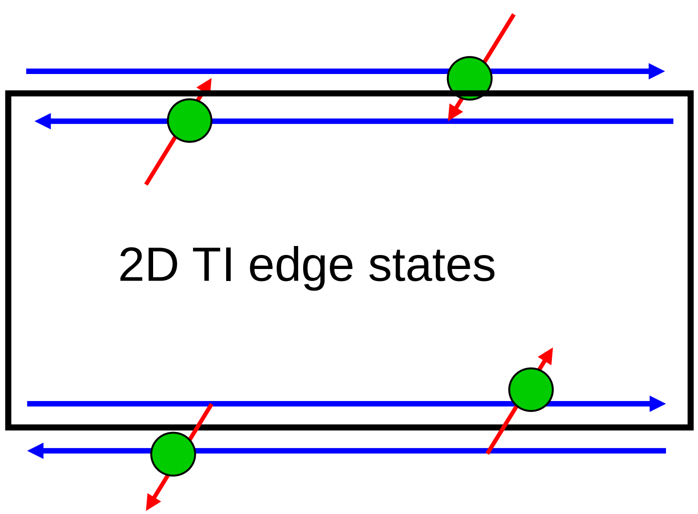
Graphene as a 2D QSH or topo-insulator (TI): tiny gap

- Kane-Mele model: spin-orbit (SO) coupling = two copies of Haldane model.



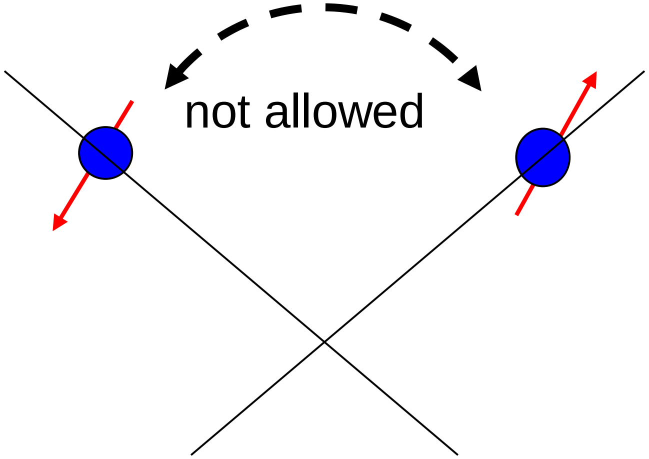
Kane and Mele, PRL95, 146802(2005); PRL 95, 226801(2005).

- Helical edge states.



Instability: the single-particle back-scattering

$$H_0 = v_f \int dx(\psi^+_{R\uparrow} i\partial_x \psi_{R\uparrow} - \psi^+_{L\downarrow} i\partial_x \psi_{L\downarrow})$$



- Kane and Mele : The non-interacting helical systems remain gapless against disorder and impurity scatterings.

- Single particle backscattering term breaks TR symmetry (T^2=-1).

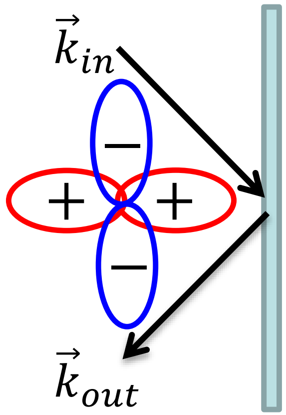
$$H_{bg} = \psi^+_{R\uparrow}\psi_{L\downarrow} + \psi^+_{L\downarrow}\psi_{R\uparrow} \qquad T^{-1}H_{bg}T = -H_{bg}$$

- How about interacting effects?

High Tc superconductivity: zero-energy Andreev boundary modes

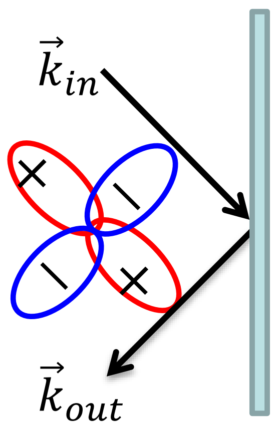
[10] boundary:

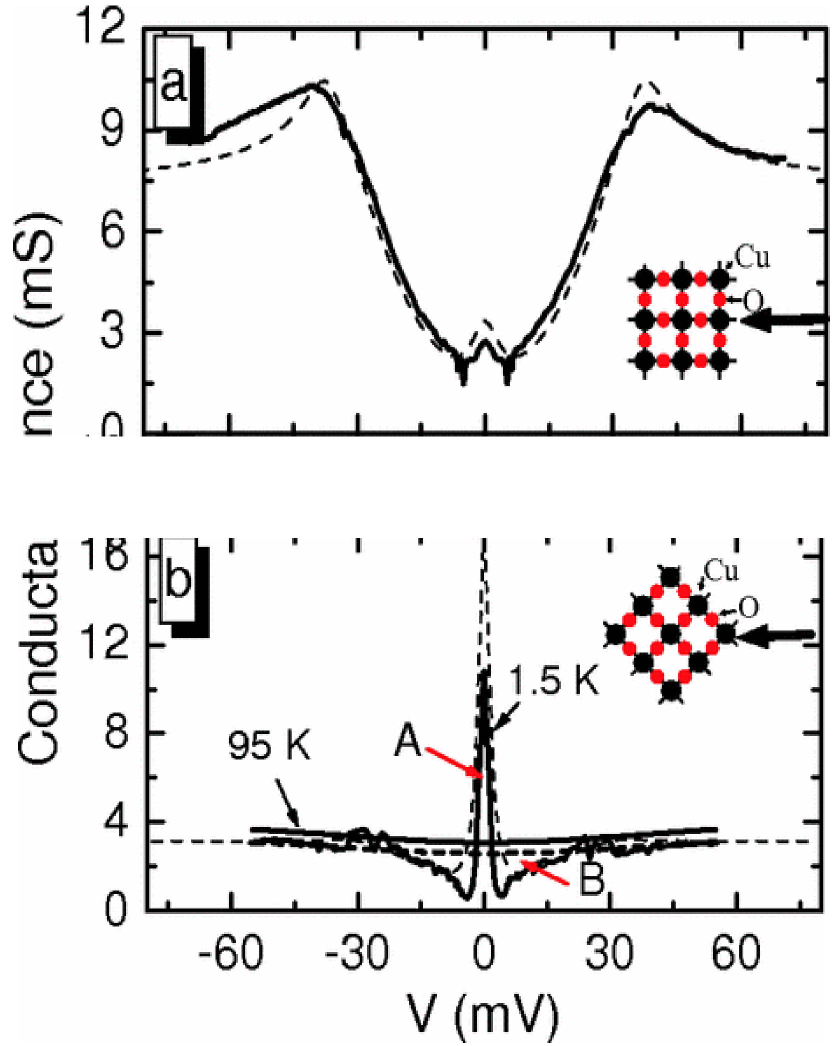
$\Delta\left(\vec{k}_{in}\right) = \Delta\left(\vec{k}_{out}\right)$



[11] boundary:

$\Delta\left(\vec{k}_{in}\right) = -\Delta\left(\vec{k}_{out}\right)$



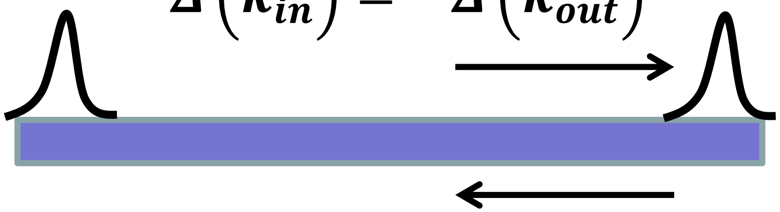


C.-R. Hu, Phys. Rev. Lett. 72, 1526 (1994)
L. H. Greene, et al, PRL 89, 177001 (2002)

Majorana boundary zero modes

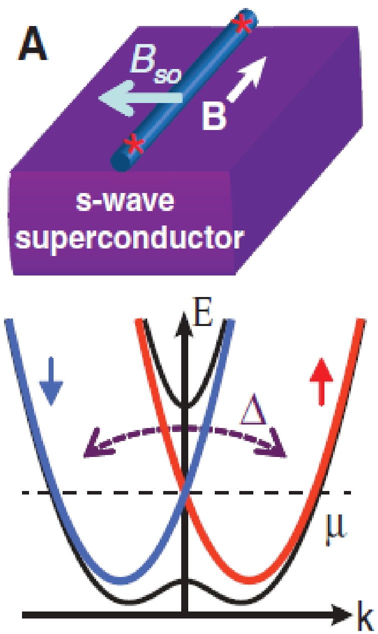
- **Single-component fermion p-wave pairing in 1D – Kitaev 2001.**

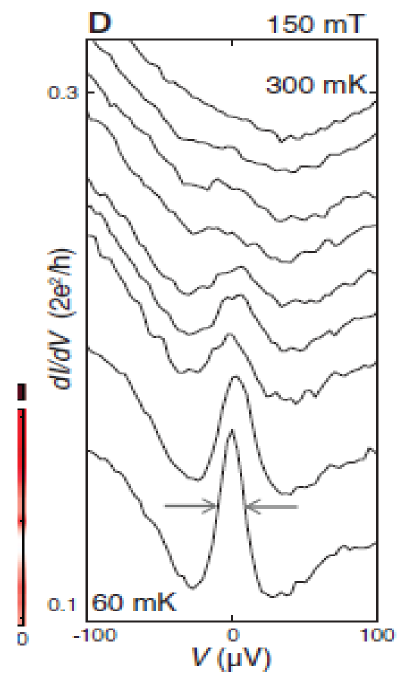
$$\Delta\left(\vec{k}_{in}\right) = -\Delta\left(\vec{k}_{out}\right)$$



$$\gamma = \int dx\, u_0(x)\psi(x) + v_0(x)\psi^{+}(x)$$

$$u_0(x) = v_0^{*}(x)$$



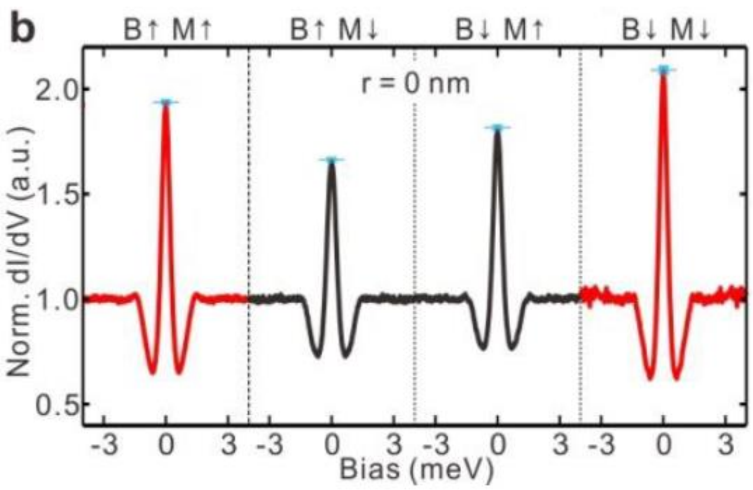


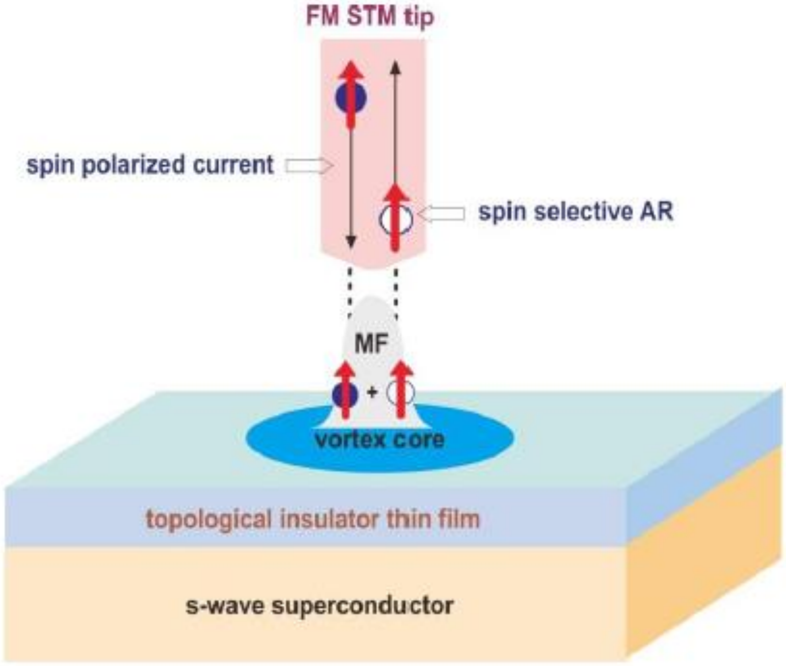
- **Spin-orbit coupled superconducting wire under magnetic field – in debate.**

P. A. Lee, arxiv 0907.2681
Sau, Lutchyn, Das Sarma PRL 2010.
Liu, Potter, Law, Lee, PRL 109, 267002 (2011);
He, Ng, Lee, Law, PRL (2014).

L. P. Kouwenhouven, et al, Science 336 1003 (2012);
C. M. Marcus et al, PRB 87, 241401 (2013).

Observation of the Majorana mode

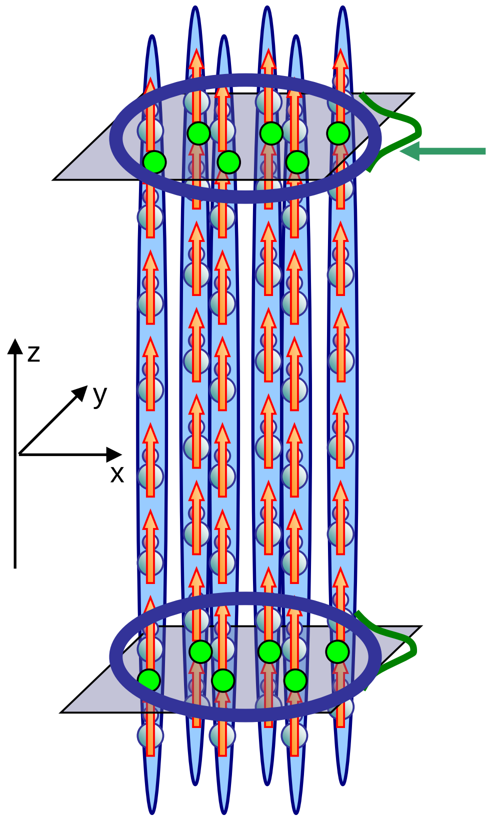




H. H. Sun, et al, PRL 116, 257003(2016)

Majorana edge modes in quasi-1D Topo superconductors

Andreev bound states localized at ends z_0 with energy zero.



$$\gamma_i = \int \mathrm{d}x\{u_0(x)\mathrm{e}^{-\mathrm{i}\frac{\theta_i}{2}-\mathrm{i}\frac{\pi}{4}}\psi_i(x) + v_0(x)\mathrm{e}^{\mathrm{i}\frac{\theta_i}{2}+\mathrm{i}\frac{\pi}{4}}\psi_i^\dagger(x)\},$$

$$u_0(x) = v_0(x) \approx \mathrm{e}^{-\frac{L-x}{\xi_x}} \sin k_\mathrm{f} x,$$

Dispersionless in k_x and k_y

Andreev Bound States
→ 1D or 2D Majorana fermions lattices.

Yi Li, Da Wang, Congjun Wu, New J. Phys 15, 085002(2013)

Kitaev, 2000;
Tewari, et al, 2007;
Alicea, et al, 2010;
etc ...

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Helical Luttinger liquid and its stability against strong interactions.

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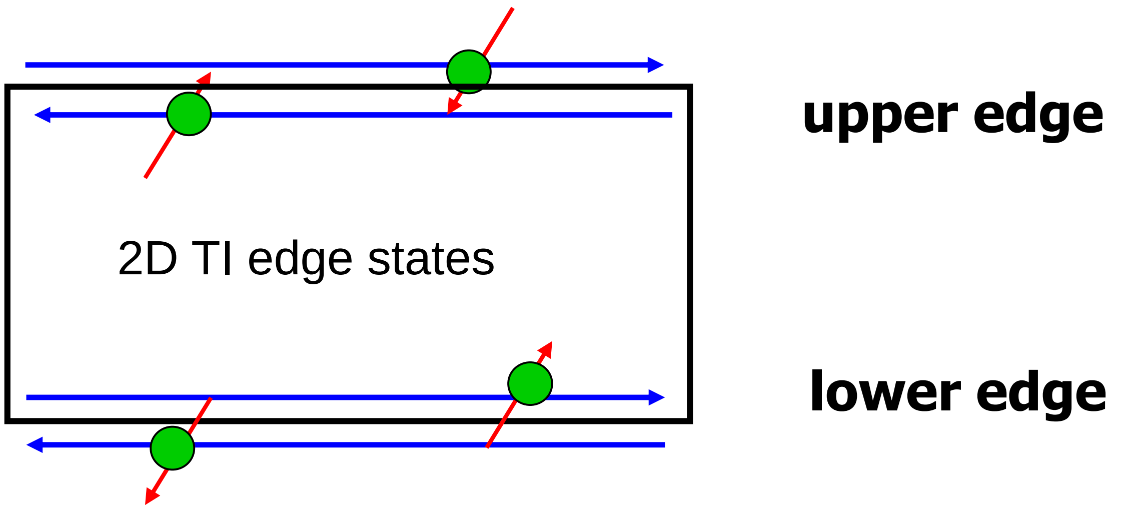
Interacting 2D TI edges: helical Luttinger liquid (HLL)



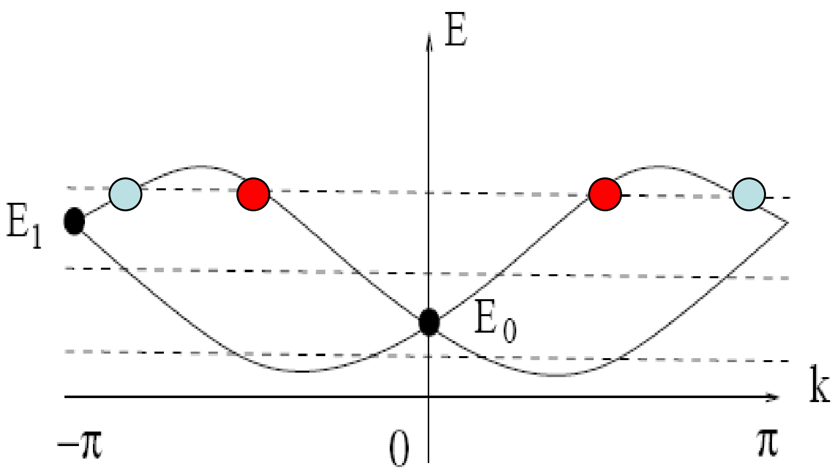
- Helical Luttinger liquid is special.

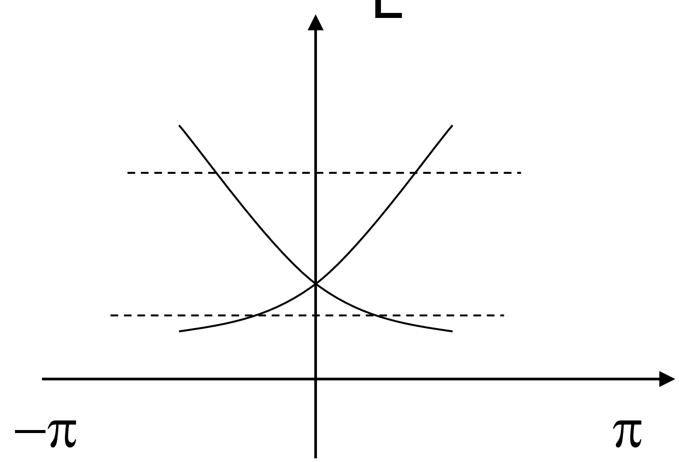
1. chiral Luttinger liquids in quantum Hall edges break TR symmetry;
2. spinless non-chiral Luttinger liquids: $T^2=1$;
3. non-chiral spinful Luttinger liquids have an even number of branches of TR pairs.

Kane et al., PRL 2005, C. Wu et al., PRL 2006.

The "no-go" theorem for helical Luttinger liquids

• 1D HLL with an odd number of components can NOT be constructed in a purely 1D lattice system.





• Double degeneracy occurs at k=0 and π.

• Periodicity of the Brillouin zone.

• HLL with an odd number of components can appear as the edge states of a 2-D system.

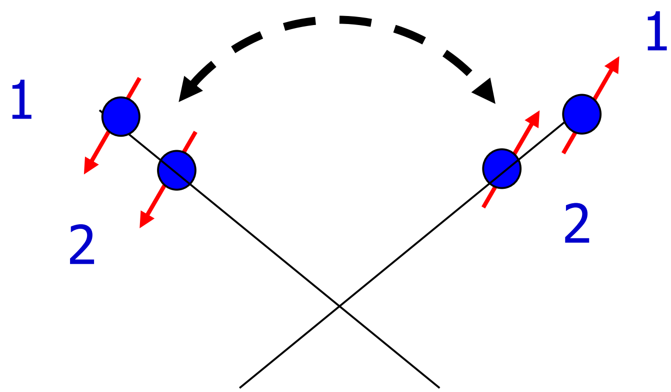
H. B. Nielsen et al., Nucl Phys. B 185, 20 (1981); C. Wu et al., Phys. Rev. Lett. 96, 106401 (2006).

Helical edge modes are stable under weak interactions, but can be destabilized by strong interactions!

C. Wu, B. A. Bernevig, S. C. Zhang, PRL 96, 1060401 (2006); C. Xu, J. Moore, PRB 2006.

Two-particle correlated back-scattering

- TR symmetry allows two-particle correlated back-scattering.



$$G_2(t,0) = \left\langle \psi_{R\uparrow 1}(t)\psi_{R\uparrow 2}(t)\ \psi^{+}_{L\downarrow 2}(0)\psi^{+}_{L\downarrow 1}(0)\right\rangle_{connected} \neq 0$$

$$H_{um} = \sum_{\langle ij \rangle} s_x(i)s_x(j) - s_y(i)s_y(j),$$

$$\text{or} \sum_{\langle ij \rangle} s_x(i)s_y(j) + s_y(i)s_x(j)$$

- Microscopically, this Umklapp process can be generated from anisotropic spin-spin interactions.

- Effective Hamiltonian: $H_{um} = g_u \int dx\, e^{i4k_f} \psi^{+}_{R\uparrow}(x)\psi^{+}_{R\uparrow}(x+\varepsilon)\psi_{L\downarrow}(x+\varepsilon)\psi_{L\downarrow}(x) + h.c.$

- U(1) rotation symmetry → Z_2. $s_x \to -s_x,\quad s_y \to -s_y,\quad s_z \to s_z$

Bosonization+Renormalization group

- Sine-Gordon theory at $k_f=\pi/2$.

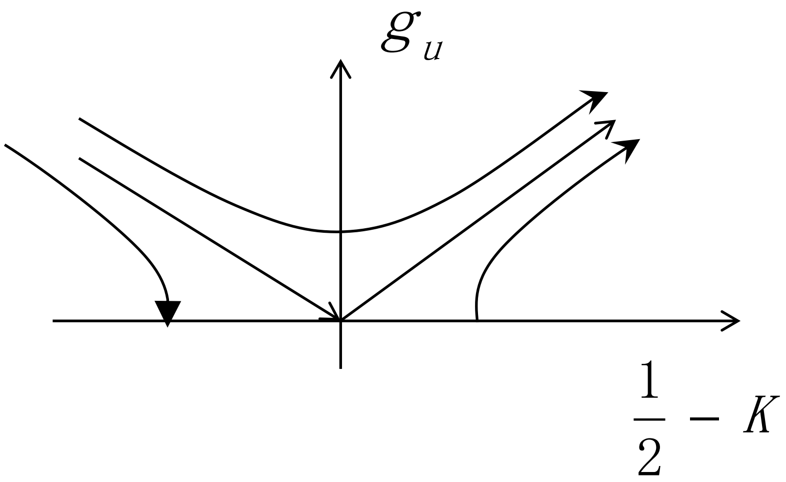
$$\psi_{R\uparrow} \propto e^{i\sqrt{4\pi}\phi_{R\uparrow}},\ \psi_{L\downarrow} \propto e^{-i\sqrt{4\pi}\phi_{L\downarrow}};\ \phi(\theta)=\phi_{R\uparrow}\pm\phi_{L\downarrow}$$

$$H_0=\int dx \frac{v}{2}\{\frac{1}{K}(\partial_x\phi)^2+K(\partial_x\theta)^2\}+\frac{g_u}{2(\pi a)^2}\cos\sqrt{16\pi}\phi$$

- The scaling dimension:

$$\Delta_{g_u}=\frac{(\sqrt{16\pi K})^2}{4\pi}=4K$$

$$\frac{dg_u}{d\ln L}=(2-\Delta_{g_u})g_u$$



Bosonization+Renormalization group

- The cosine term is relevant at K<1/2 (strong repulsion); gap opens with

$$g_u \to \pm\infty \qquad \left\langle \cos\sqrt{16\pi}\phi \right\rangle \neq 0 \qquad \Delta \approx a^{-1}(g_u)^{\frac{1}{4(1/2-K)}}$$

- Order parameters $2k_f$ SDW orders N_x (g_u<0) or N_y at (g_u>0) .

$$N_x \propto \cos\sqrt{4\pi}\phi, \qquad N_y \propto \sin\sqrt{4\pi}\phi$$

- TR symmetry is spontaneously broken in the ground state.

- At $\Delta >> T > 0\mathrm{K}$, TR symmetry must be restored by thermal fluctuations and the gap → pseudo gap.

Random two-particle back-scattering

- Scattering amplitudes $g_u(x)e^{i\alpha(x)}$ are quenched Gaussian variables.

$$H_{\text{int}} = \int dx \frac{g_u(x)}{2(\pi a)^2} \cos(\sqrt{16\pi}\phi + \alpha(x))$$

$$\left\langle g_u(x)e^{i\alpha(x)}\; g_u(y)e^{-i\alpha(y)} \right\rangle = D\delta(x-y) \qquad \frac{dD}{d\ln t} = (3-8K)D$$

Giamarchi, *Quantum physics in one dimension,* oxford press (2004).

- If K<3/8, gap D opens. SDW order is spatially disordered but static in the time domain.

- At small but finite temperatures, gap remains but TR is restored by thermal fluctuations.

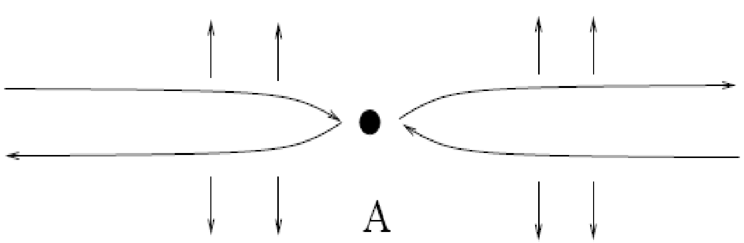
Single impurity scattering

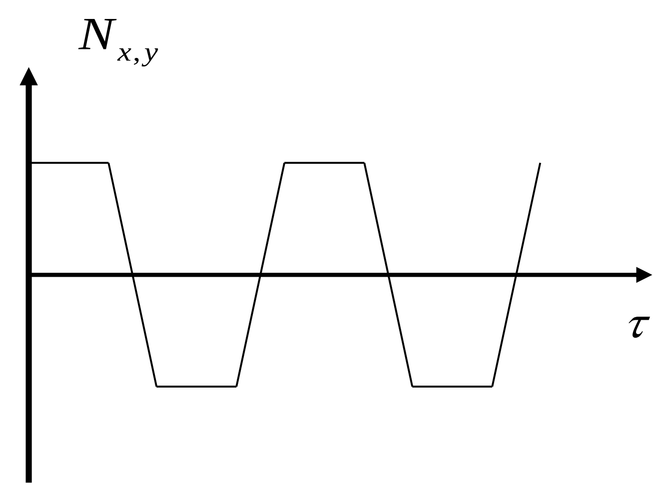
- Boundary Sine-Gordon equation.

$$H_{\text{int}} = \int dx \frac{g_u}{2(\pi a)^2} \delta(x) \cos(\sqrt{16\pi}\phi) \qquad \frac{dg_u}{d \ln t} = (1-4K) g_u$$

C. Kane and M. P. A. Fisher, PRB 46, 15233 (1992).

- If K<1/4, g_u term is relevant. 1D line is divided into two segments.



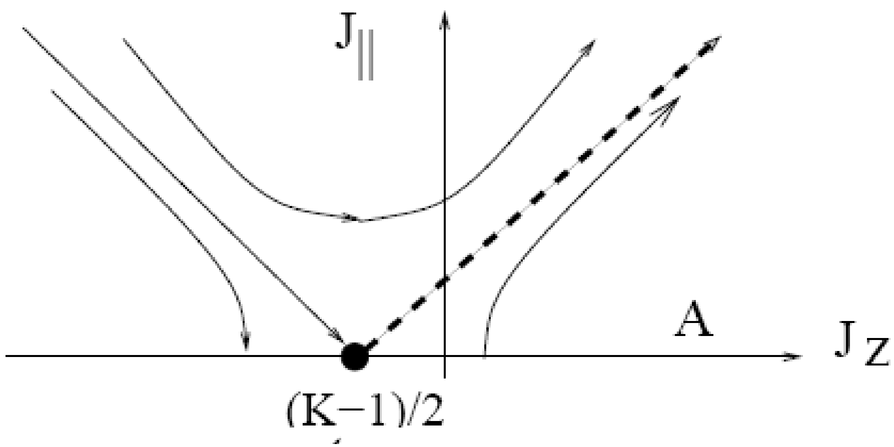


$$N_x \propto \cos\sqrt{4\pi}\phi,\ N_y \propto \sin\sqrt{4\pi}\phi$$

Kondo problem: magnetic impurity scattering

$$H_K = \int dx\, \delta(x)\{\frac{J_{//}}{2}(\sigma_- \psi^+_{R\uparrow}\psi_{L\downarrow} + \sigma_+ \psi^+_{L\downarrow}\psi_{R\uparrow}) + J_z \sigma_z (\psi^+_{R\uparrow}\psi_{R\uparrow} - \psi^+_{L\downarrow}\psi_{L\downarrow})\}$$

- Poor man RG: critical coupling J_z is shifted by interactions.
- If K<1 (repulsive interaction), the Kondo singlet can form with ferromagnetic couplings.



$$\frac{dJ_z}{d\ln t} = 2J_{//}^2,$$

$$\frac{dJ_z}{d\ln t} = (1 - K + 2J_z)J_{//}$$

Instability criterial for helical Luttinger liquid

- Two-particle correlated back-scattering is allowed by TR symmetry, and becomes relevant at:

  $K_c<1/2$ for Umklapp scattering at commensurate fillings.

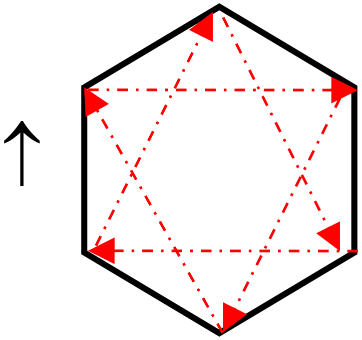
  $K_c<3/8$ for disordered two-particle scattering.

  $K_c<1/4$ for a single impurity two-particle scattering.

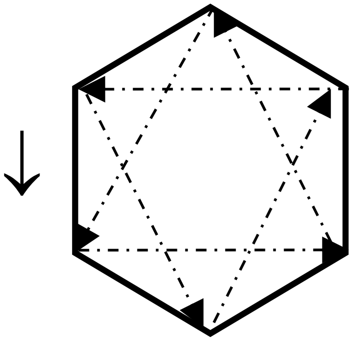
- The Kondo critical point is shifted by interaction.

The Kane-Mele-Hubbard (KMH) model

- Double of the Haldane's model quantum anomalous Hall model: spin up and down components with opposite flux patterns.



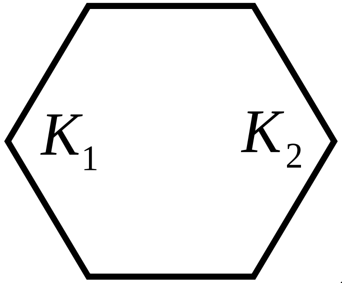
$$H_{NN} = -t\sum_{\vec{r}\in A}\{c_{\alpha}^{+}(\vec{r}_A)c_{\alpha}(\vec{r}_B) + h.c.\}$$



$$H_{NNN} = \sum_{\vec{r}\vec{r}'} t' e^{i\delta}\{c_{\uparrow}^{+}(\vec{r}_A)c_{\uparrow}(\vec{r}_A') + c_{\uparrow}^{+}(\vec{r}_B)c_{\uparrow}(\vec{r}_B')\}$$
$$+ \sum_{\vec{r}\vec{r}'} t' e^{-i\delta}\{c_{\downarrow}^{+}(\vec{r}_A)c_{\downarrow}(\vec{r}_A') + c_{\downarrow}^{+}(\vec{r}_B)c_{\downarrow}(\vec{r}_B')\} + h.c.$$

- The Hubbard interaction.

$$H_U = U\sum_{\vec{r}}(n_{\uparrow}(\vec{r}) - \tfrac{1}{2})(n_{\downarrow}(\vec{r}) - \tfrac{1}{2})$$



Monte-Carlo (Aesthetics of brutal force：大巧不工)

- Probability distribution:

$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

$$w(\{\sigma_i\}) = \exp[-\beta H(\{\sigma_i\})] / Z$$

- Observables: magnetization and susceptibility.

$$M = \frac{1}{N} \sum_{\{\sigma\}} w\{\sigma\} (\sum_i \sigma_i) \qquad \chi = \frac{1}{N^2} \sum_{\{\sigma\}} w\{\sigma\} (\sum_i \sigma_i)^2$$

- Importance sampling and detailed balance (Metropolis):

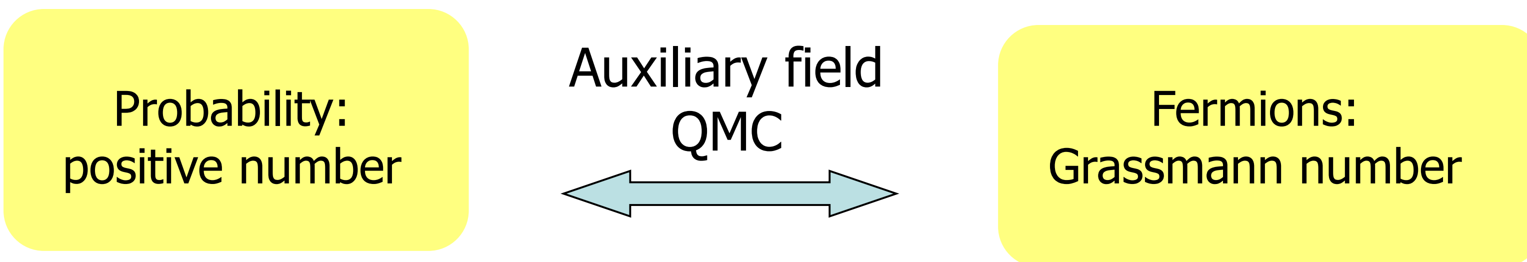
1. Start from a configuration {s} with probability w({s}).
   Get a trial configuration by flipping a spin.
2. Calculate acceptance ratio: $r = w(\{\sigma_{trial}\}) / w(\{\sigma\})$.
3. If r>1, accept it; If r<1, accept it wit the probability of r.

Auxiliary field QMC for fermions

Blankenbecler, Scalapino, and Sugar. PRD 24, 2278 (1981)

- Using path integral formalism, fermions are represented as Grassmann variables.

- Transform Grassmann variables into probability.



- Decouple interaction terms using Hubbard-Stratonovich (H-S) bosonic fields.

- Integrate out fermions and the resulting fermion functional determinants work as statistical weights.

The sign (phase) problem!!!

• Generally, the fermion functional determinants are not positive definite. Sampling with the absolute value of fermion functional determinants.

$$\langle O \rangle = \langle\langle \mathrm{sign} \times O \rangle\rangle \Big/ \langle\langle \mathrm{sign} \rangle\rangle$$

• Huge cancellation in the average of signs.

• Statistical errors scale exponentially with the inverse of temperatures and the size of samples.

• Finite size scaling and low temperature physics inaccessible.

Absence of the sign problem in the KMH model

- Projector method (T=0K).
- H-S decomposition. P-h transf. one spin component.

$$\langle \hat{O} \rangle = \frac{\langle \psi_0 | \hat{O} | \psi_0 \rangle}{\langle \psi_0 | \psi_0 \rangle} = \frac{\langle \psi_T | e^{-\Theta H} \hat{O} e^{-\Theta H} | \psi_T \rangle}{\langle \psi_T | e^{-2\Theta H} | \psi_T \rangle}$$

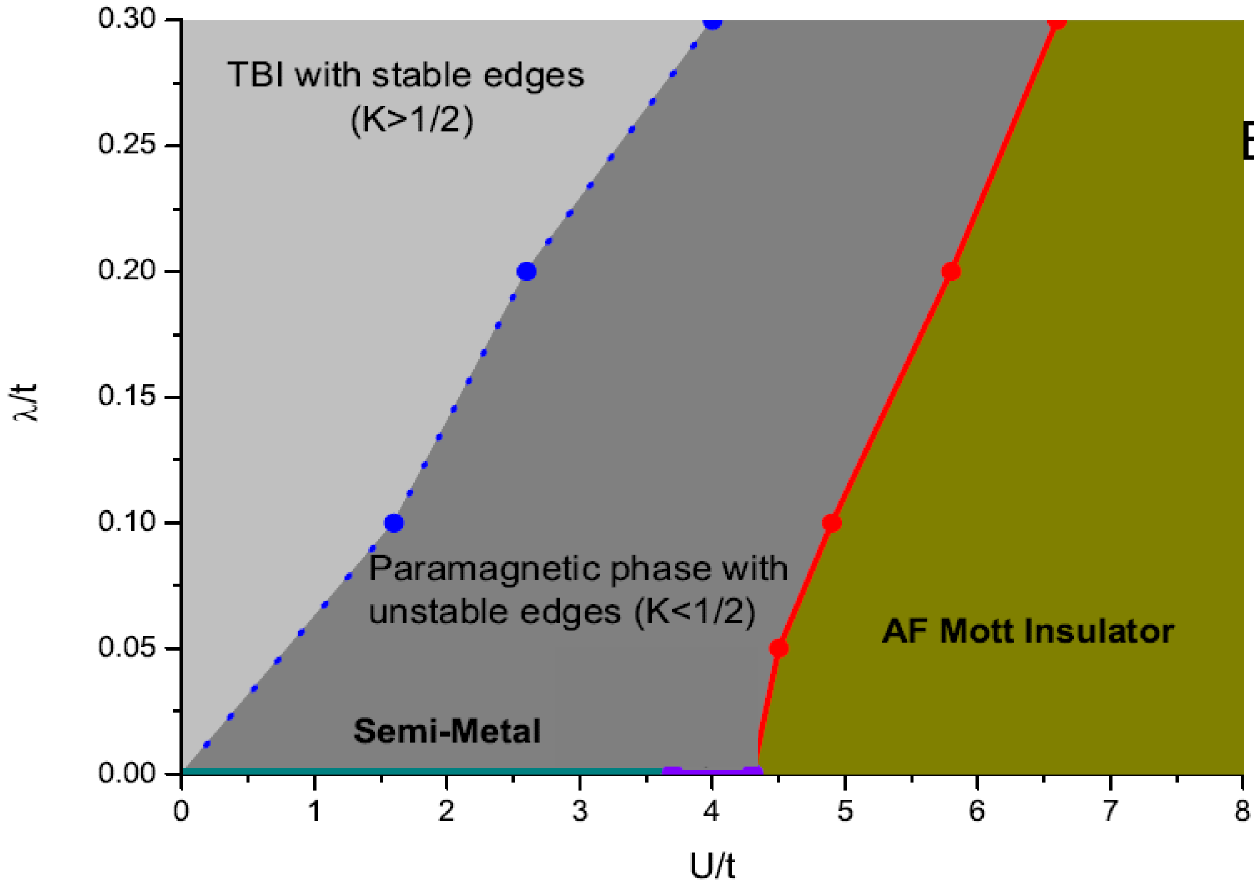
$$e^{-\Theta H} = \sum_{\{l\}} \{ \prod_{p=M}^{1} e^{-\Delta\tau \sum_{i,j} c^{+}_{i\uparrow} K^{\uparrow}_{ij} c_{j\uparrow}} e^{i\sqrt{\Delta\tau U/2} \sum_{i} \eta_{i,p} (c^{+}_{i\uparrow} c_{i\uparrow} - 1/2)} \prod_{p=M}^{1} e^{-\Delta\tau \sum_{i,j} d^{+}_{i\downarrow} K^{\downarrow}_{ij} d_{j\downarrow}} e^{-i\sqrt{\Delta\tau U/2} \sum_{i} \eta_{i,p} (d^{+}_{i\downarrow} d_{i\downarrow} - 1/2)} \prod_{i,p} \gamma_{i,p}(l) \}$$

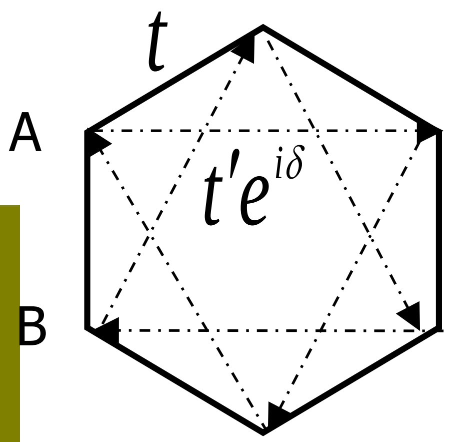
- Determinants factorize into complex conjugate pairs → positive definite statistical weight.

$$\mathrm{Det}(\uparrow) = \mathrm{Det}^{*}(\downarrow)$$

The global phase diagram of KMH model at half-filling

- Sign-problem free QMC.



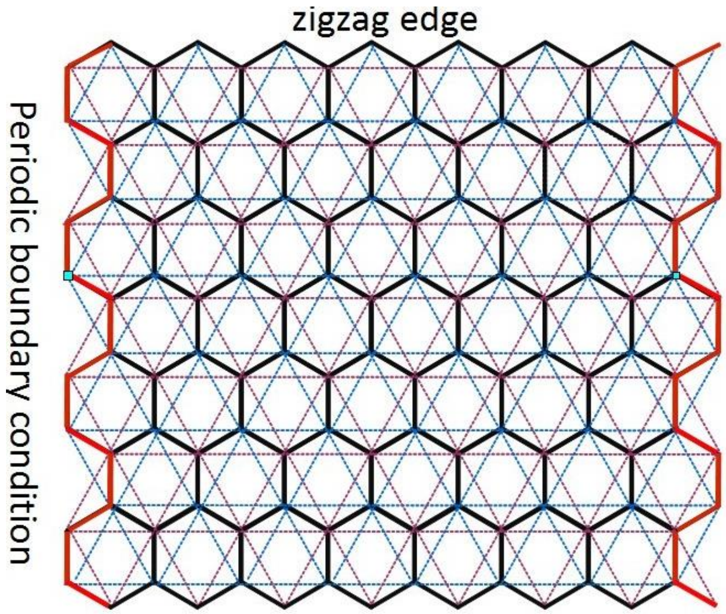


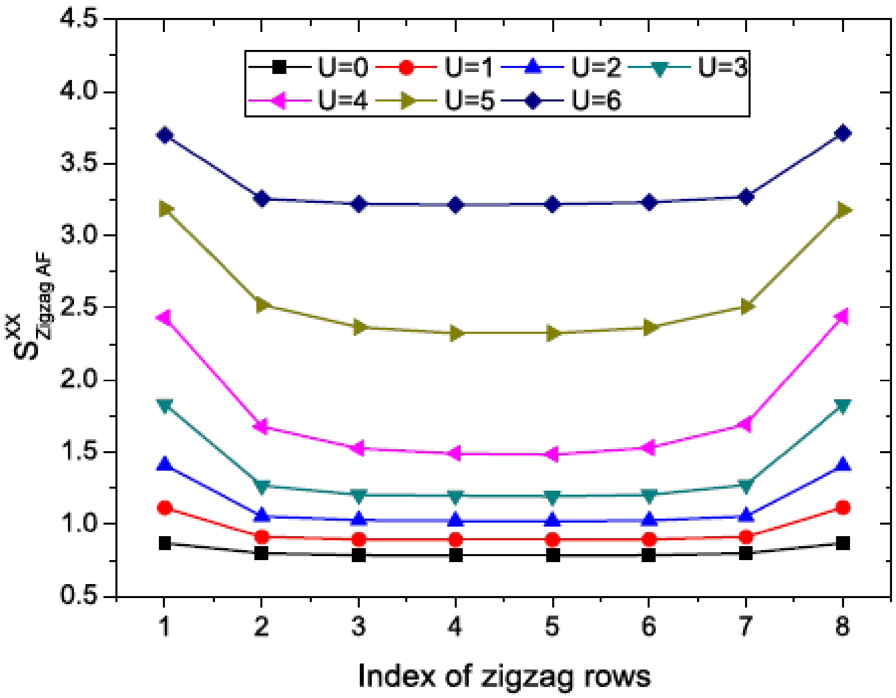
Dong Zheng, Guang-Ming Zhang, C. Wu, Phys. Rev. B 84, 205121 (2011).

- A large region with paramagnetic bulk but antiferromagnetic edge states.

Edge properties with non-magnetic bulk

- AF form factor for each zig-zag line from the edge to the center.
- AF is weakest in the middle and strongest at the edge.





8*8*2; x periodical; y: open

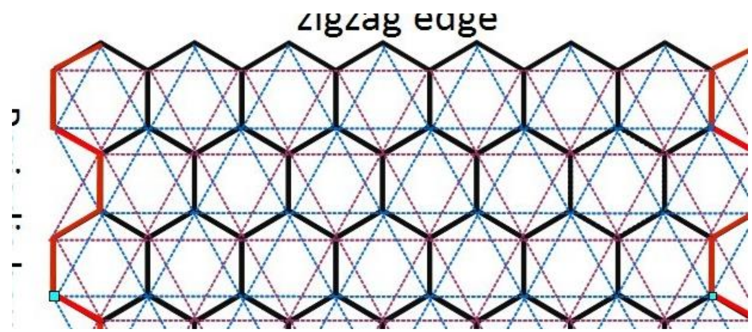
Zheng, G. M. Zhang, C. Wu PRB 84, 205121 (2011) .

$\lambda = 0.1, U_c \approx 5$

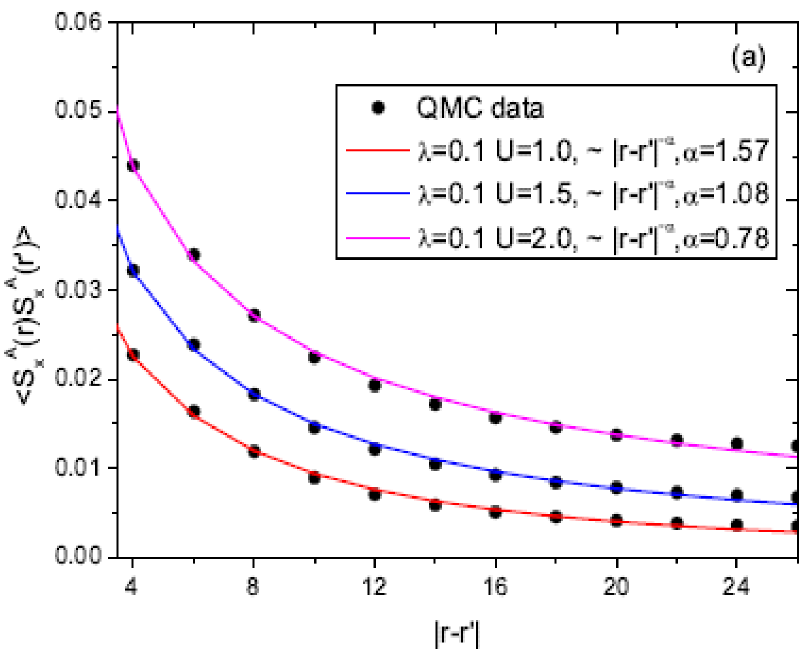
AF two-point correlations along the edge

$$S_x + iS_y = \psi^{+}_{R\uparrow}\psi_{L\downarrow} \propto e^{i\sqrt{4\pi}\phi}$$

$$\left\langle S_x(x)S_x(x')\right\rangle \sim 1/\,|\,x - x'\,|^{-2K}$$



A narrow strip 34*4*2



- The two different phases according to K<(>) 1/2.

$$K = 0.8 \qquad U = 1$$

$$0.5 \qquad U = 1.5$$

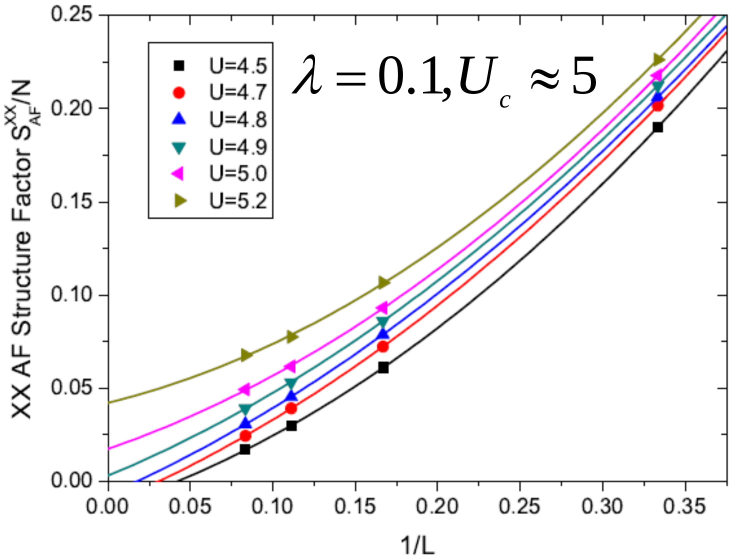
$$0.4 \qquad U = 2$$

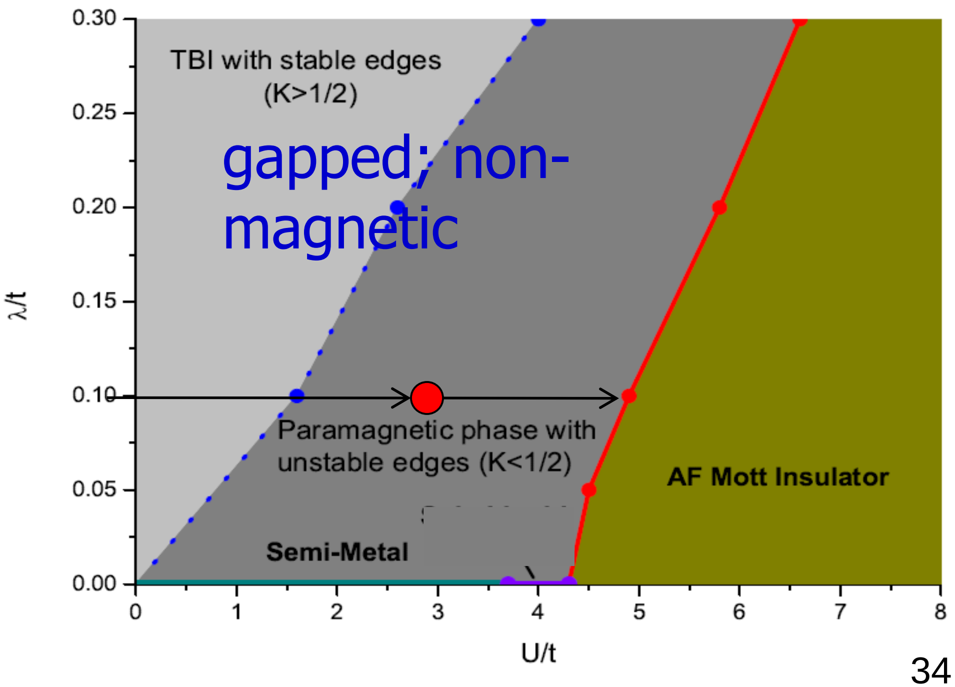
Large U: bulk antiferromagnetism (AF)

- Easy plane antiferromagnet.

NN: $\frac{4t^2}{U}\vec{S}\cdot\vec{S}$ NNN: $\frac{4\lambda^2}{U}(-S_x \cdot S_x - S_x \cdot S_x + S_z \cdot S_z)$

- AF form factor: $S^{xx}_{AF} = \frac{1}{N}\langle G| \left[\sum_i (-1)^i S^x_i\right]^2 |G\rangle,$





Meng et al., Nature 2010. Hohenadler, Lang, and Assaad arxiv:1011.5063; Zheng, Wu, G. M. Zhang, arxiv:1011.5858.

Outline

• Introduction.

Quantum Hall → Quantum anomalous Hall → topological insulators (TI)

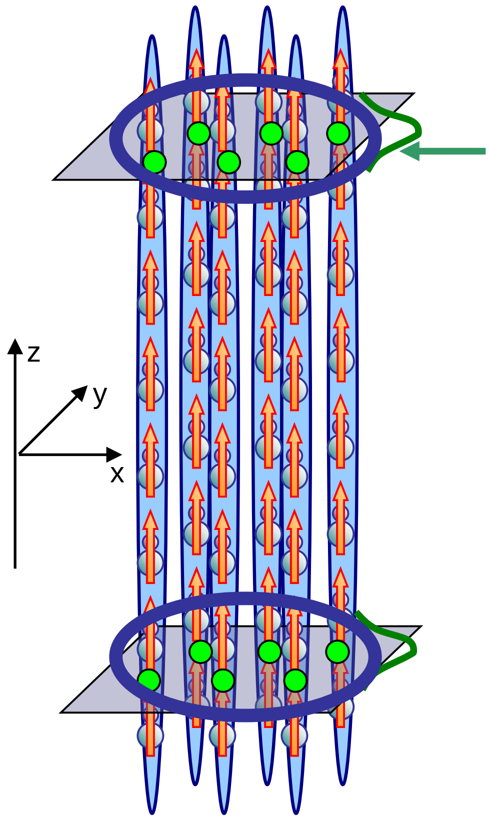
• Stability criterial of the interacting edge states of 2D TIs, and QMC study of interacting 2D TIs.

Strong interactions can gap out the helical edge states.

• Spontaneous time-reversal symmetry breaking in Majorana flat bands.

Majorana edge modes in quasi-1D Topo superconductors

Andreev bound states localized at ends z_0 with energy zero.



$$\gamma_i = \int \mathrm{d}x \{u_0(x) \mathrm{e}^{-\mathrm{i}\frac{\theta_i}{2} - \mathrm{i}\frac{\pi}{4}} \psi_i(x) + v_0(x) \mathrm{e}^{\mathrm{i}\frac{\theta_i}{2} + \mathrm{i}\frac{\pi}{4}} \psi_i^\dagger(x)\},$$

$$u_0(x) = v_0(x) \approx \mathrm{e}^{-\frac{L-x}{\xi_x}} \sin k_f x,$$

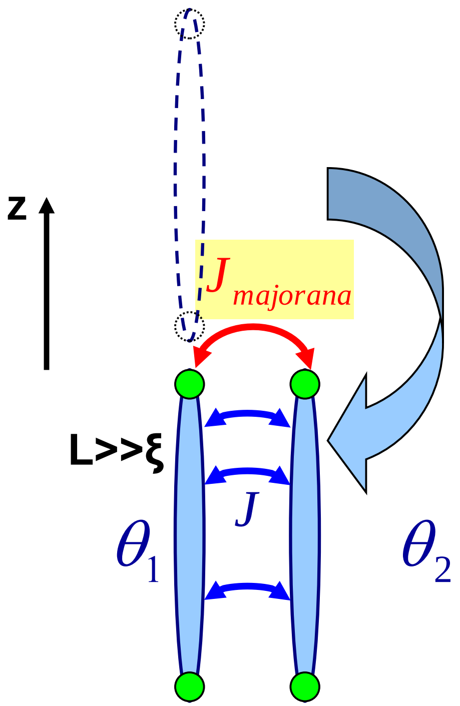
Dispersionless in k_x and k_y

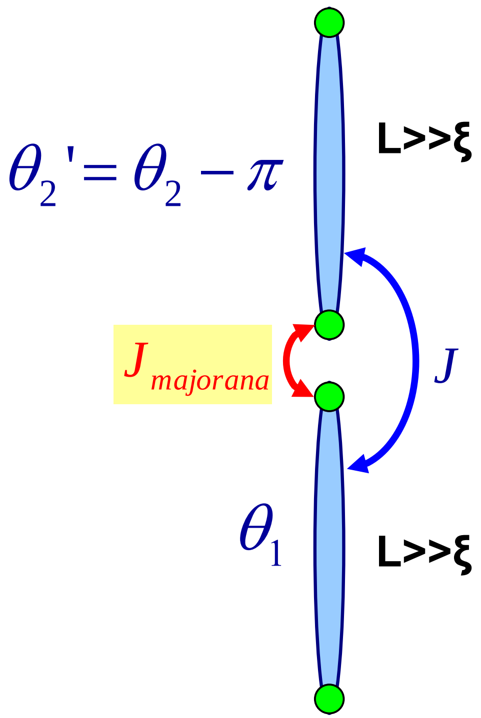
Andreev Bound States
→ 1D or 2D Majorana fermions lattices.

Kitaev, 2000;
Tewari, et al, 2007;
Alicea, et al, 2010;
etc ...

Yi Li, Da Wang, Congjun Wu, New J. Phys 15, 085002(2013)

Majorana Josephson coupling between chains





leading order

$$H_t = -J\cos(\theta_1 - \theta_2)$$

$$- J_{majorana} i\gamma_1\gamma_2 \underline{\sin}(\frac{\theta_1 - \theta_2}{2})$$

$$H_t = -J\cos(\theta_1 - \theta_2')$$

$$- J_{majorana} i\gamma_1\gamma_2 \cos(\frac{\theta_1 - \theta_2'}{2})$$

[Kitaev, 2000; Yakovenko et al, 2004; Fu and Kane, 2009; Xu and Fu, 2010]

Superconducting phase – Majorana fermion coupling

$$H_t = -J \sum_{<i,j>} \cos(\theta_i - \theta_j) - iJ_{majorana} \sum_{<i,j>} \sin(\frac{\theta_i - \theta_j}{2}) \gamma_i \gamma_j$$

- Possibility (I): Uniform phase, TR symmetry maintained.

Majorana edge modes decouple – flat edge-bands.

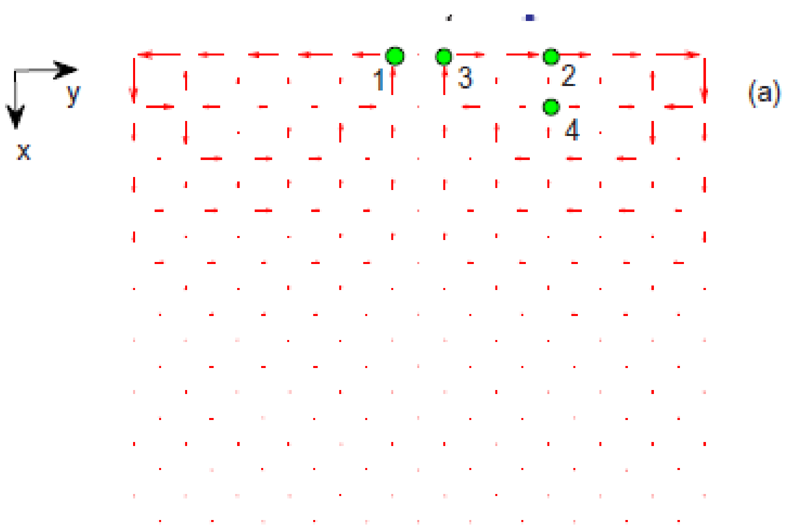
But density of states diverges → intrinsic instability!!

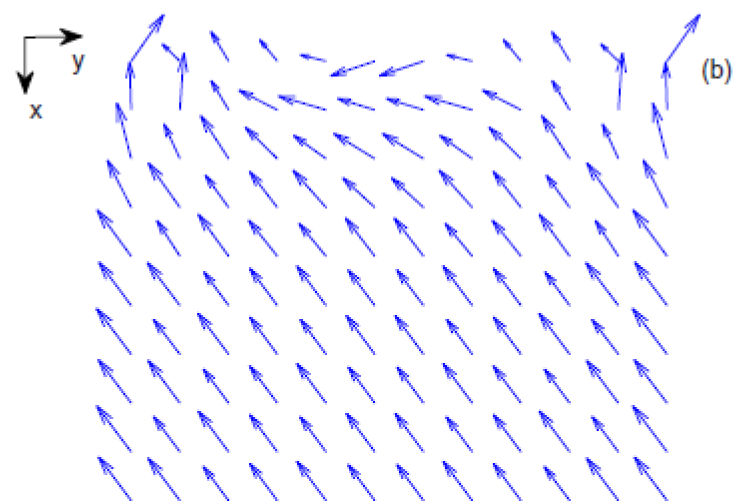
- Possibility (II): Spontaneous TR symmetry breaking.

Majorana modes coupled and develop dispersion – lowering energy.

A self-consistent calculation (B-de G)

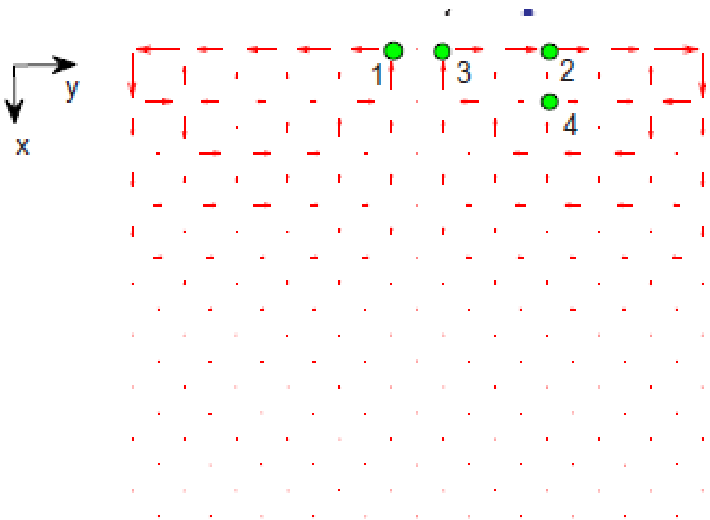
$$H_{\text{mf}} = -\sum_i \left\{ (t_x c_i^\dagger c_{i+\hat{e}_x} + t_y c_i^\dagger c_{i+\hat{e}_y} + \text{h.c.}) - \mu c_i^\dagger c_i \right\} - V \sum_i \left\{ \Delta^*_{i,i+\hat{e}_x} c_{i+\hat{e}_x} c_i + \text{h.c.} \right\}$$
$$+ V \sum_i \Delta^*_{i,i+\hat{e}_x} \Delta_{i,i+\hat{e}_x} ,$$

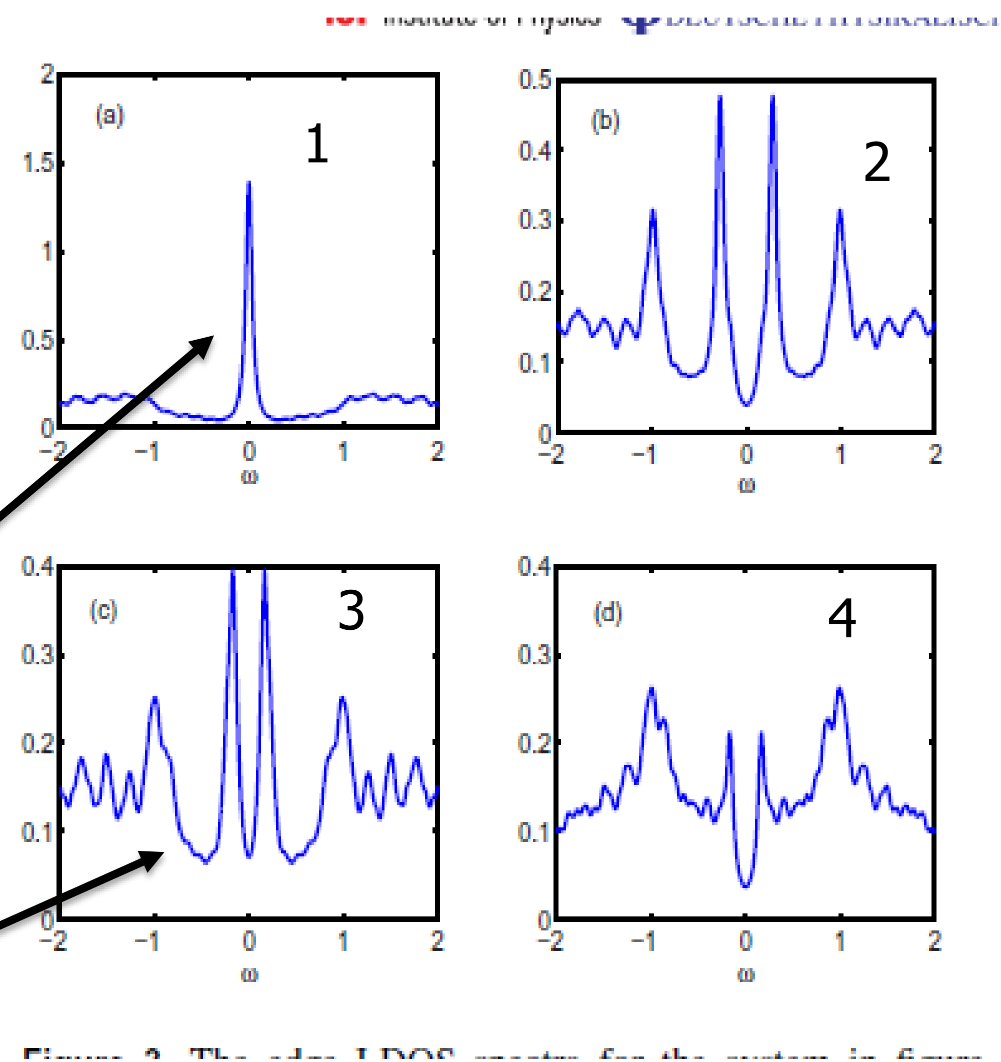




- Supercurrent distribution – non-quantized vortex-antivortex
- Superfluid phase distribution

Local Density of States (LDOS)





Non-interacting result

Interaction treated at self-consistent mean-field level

Summary

• No-go theorem: HLL with odd number of components can not be constructed in a purely 1D lattice system.

• Helical edge states are stable at weak interactions but are gapped out with increasing interactions.

• Sign-problem free simulation on Kane-Mele-Hubbard model --

Magnetic fluctuations are strongest on the edge; bulk paramagnetism + edge AFM

• Spontaneous vortex-antivortex formation in Majorana edge flat bands.